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PCELG光束通过光阑传输后偏振度的研究

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摘要: 为了研究部分相干复宗量拉盖尔-高斯(PCELG)光束通过环形光阑后在湍流大气中传输时的偏振特性,基于拓展的惠更斯-菲涅耳原理,经理论推导,得出PCELG光束偏振度的解析表达式,并进行了相应的数值计算。结果表明,光阑的衍射效应将会造成PCELG光束偏振度振荡加剧的现象;当PCELG光束在湍流大气或自由空间中传输时,近轴处横向的偏振度有着剧烈的振荡,而在较远点处,其偏振度都是趋于1,且随着传输距离的增加,横向的偏振度趋于1时所对应的离轴距离也增大;当PCELG光束通过光阑后在自由空间中传输时,经过足够长的距离,偏振度将趋近于不同于初始偏振度的值,而在湍流大气中传输时,偏振度将趋近于初始偏振度。该结果对光束在大气中的传输及其应用具有实际意义。

关键词: 大气与海洋光学;部分相干复宗量拉盖尔-高斯光束;偏振度;拓展的惠更斯-菲涅耳原理;环形光阑
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Research of degree of polarization of PCELG beam propagating through a circular aperture

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Abstract: To study the degree of polarization of partially coherent elegant Laguerre-Gaussian (PCELG) beam propagating through a circular aperture in turbulent atmosphere, based on extended Huygens-Fresnel principle, the formulas for the degree of polarization of PCELG beam were derived via theoretical calculation and the corresponding numerical calculation was carried out. The results indicate that the aperture diffraction effect will result in increasing oscillations of the polarization of PCELG beam. In turbulent atmosphere or in free space, the transverse degree of polarization near to the axis has a dramatic oscillation while the degree of polarization far from the axis approaches to 1. The further the propagation distance is, the larger the off-axis distance is. After propagating for a sufficiently long distance through the circular aperture in free space, the degree of polarization of PCELG beam approaches a value different from the initial one. However, it tends to the initial value in turbulent atmosphere. The results are useful for beam propagation and applications in atmosphere.

Key words: atmospheric and ocean optics; partially coherent elegant Laguerre-Gaussian beam; degree of polarization; extended Huygens-Fresnel principle; circular aperture

引　言

激光束在大气中传输的理论研究对激光通信、雷达和激光武器等领域有重要意义,近年来,各种激光光束在湍流大气中的传输受到了人们的广泛关注[1-2]。大量的研究表明,由于大气湍流会引起完全相干光的迅速扩展,使激光在遥感、跟踪和远距离光通信方面上的应用有一定的局限性。相对于完全相干光而言,部分相干光在湍流大气中传输具有一定的优越性[3-5]。光束在自由空间和湍流大气中传输时,其偏振度含有大量的信息,因此,研究部分相干光在自由空间和湍流大气中传输的偏振特性是非常有意义的[6-22]。

在实际工作中,研究电磁光束通过光阑时的偏振特性具有非常重要的价值。ZHAO等人基于相干性和偏振性统一理论,研究了随机电磁光束在湍流

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大气中通过光阑的偏振调制，并探索了通过改变截断参量来调节光束偏振度的方法[9]。PAN 采用光束相干-偏振矩阵方法对部分偏振高斯-谢尔模型光束被光阑衍射时的远场特性进行了研究，分析论证了远场的偏振特性与光阑的截断参量、空间相干性以及场点位置衍射角之间的关系[23]。WANG 等人研究了随机电磁高斯-谢尔模型光束在湍流大气中通过光阑的偏振特性，得出了结论：光束在湍流大气和自由空间中经过有效传播距离后，沿小孔方向和垂直于小孔方向的偏振度是一致的[24]。JI 等人研究了湍流大气对受光阑限制的电磁高斯谢尔模型光束的影响，经过足够长的传播距离后，具有不同截断参量和光束相干参量的受限电磁高斯谢尔模型光束的偏振度趋向于源平面上的偏振度[25]。ZHAO 对高斯谢尔模型光束经光阑衍射后的轴向偏振特性进行了理论研究和数值分析[26]。SHU 等人基于随机电磁光束的相干偏振统一理论和部分相干光的传输定律，研究了部分相干光源所发出的光经过多个圆孔衍射后的偏振度变化情况[27]。

本文中基于相干性和偏振性统一理论，采用 Rytov 近似处理湍流的方法，通过将光阑函数展开为有限项复高斯函数之和的形式[28]，推导了部分相干复宗量拉盖尔-高斯（partially coherent elegant Laguerre-Gaussian，PCELG）光束在湍流中的交叉谱密度矩阵元的解析表达式，进而得到了该光束的偏振度解析表达式，最后进行了相应的数值计算和分析。

1　理论模型

傍轴近似条件下，PCELG 光束在源平面（$z=0$）的电场表达式[29-31]为：

$$E_0(r,\theta,0) = A\left(\frac{r}{w_0}\right)^m \mathrm{L}_n^m\left(\frac{r^2}{{w_0}^2}\right)\times \exp\left(-\frac{r^2}{{w_0}^2}\right)\exp(\mathrm{i}m\theta) \tag{1}$$

式中，w_0 为源场中高斯光束的束腰宽度，r 和 θ 分别表示柱坐标系中径向半径与角度，$\mathrm{L}_n^m\left(\frac{r^2}{{w_0}^2}\right)$代表自变量为 $r^2/{w_0}^2$ 的拉盖尔多项式，m 和 n 分别为径向和角项系数。令 $\rho = r/w_0$，利用公式[32]：

$$\mathrm{e}^{\mathrm{i}m\theta}\rho^m \mathrm{L}_n^m(\rho^2) = \frac{(-1)^n}{2^{2n+m}\cdot n!}\times \sum_{t=0}^{n}\sum_{s=0}^{m}\mathrm{i}^s\begin{bmatrix} n \\ t\end{bmatrix}\begin{bmatrix} m \\ s\end{bmatrix}\mathrm{H}_{2t+m-s}(x)\mathrm{H}_{2n-2t+s}(y) \tag{2}$$

式中，t 为系数，s 为方次，x 和 y 为变量，H 为厄米多项式，i 为虚数单位，把（2）式代入（1）式，可得到直角坐标系下源平面 PCELG 光束的场强表达式：

$$E_0(\rho',0) = A\frac{(-1)^n}{2^{2n+m}\cdot n!}\sum_{t=0}^{n}\sum_{s=0}^{m}\mathrm{i}^s\begin{bmatrix} n \\ t\end{bmatrix}\begin{bmatrix} m \\ s\end{bmatrix}\times \mathrm{H}_{2t+m-s}\left(\frac{x}{w_0}\right)\mathrm{H}_{2n-2t+s}\left(\frac{y}{w_0}\right)\exp\left(-\frac{x^2+y^2}{{w_0}^2}\right) \tag{3}$$

基于相干性和偏振性统一理论，在源平面（$z=0$），PCELG 光源的交叉谱密度矩阵元可表示为[33-34]：

$$W_{ij,0}(\rho_1',\rho_2',\omega) = \sqrt{I_{i,0}(\rho_1',\omega)I_{j,0}(\rho_2',\omega)}\,\eta_{ij,0}(\rho_2'-\rho_1',\omega) \tag{4}$$

式中，ω 表示频率，$I_0(\rho_1',\omega)$为部分相干光束在源平面的强度分布，$\eta_{ij,0}(\rho_2'-\rho_1',\omega)$表示源平面电场分量 E_i 和 E_j 的相干度[35]，且可以表示为 $\eta_{ij,0}(\rho_1'-\rho_2',\omega)=B_{ij}\exp[-(\rho_1'-\rho_2')^2/(2{\delta_{ij}}^2)]$，其中 $i=x,y;j=x,y$。$0\leqslant\eta_{ij,0}\leqslant 1$，$\eta_{ij,0}=1$ 时表示完全相干，$\eta_{ij,0}=0$ 时表示完全非相干，$0<\eta_{ij,0}<1$ 时表示部分相干，δ_{ij}是自相干长度（$i=j$）或互相干长度（$i\neq j$），B_{ij}是一个与频率 ω 有关、而与位置无关的常数。

$$\begin{cases} B_{ij} = 1,(i=j) \\ B_{ij}\leqslant 1,(i\neq j)\end{cases} \tag{5}$$

为了计算方便，不妨假设 $w_{0i}=w_{0j}=w_0$，经计算可得，PCELG 光束源平面的交叉谱密度为：

$$W_{ij,0}(\rho_1',\rho_2',0) = \frac{A_iA_jB_{ij}}{2^{4n+2m}(n!)^2}\sum_{t_1=0}^{n}\sum_{s_1=0}^{m}\sum_{t_2=0}^{n}\sum_{s_2=0}^{m}\mathrm{i}^{s_1}(-\mathrm{i})^{s_2}\times \begin{bmatrix} n \\ t_1\end{bmatrix}\begin{bmatrix} m \\ s_1\end{bmatrix}\begin{bmatrix} n \\ t_2\end{bmatrix}\begin{bmatrix} m \\ s_2\end{bmatrix}\mathrm{H}_{2t_1+m-s_1}\left(\frac{x_1'}{w_0}\right)\mathrm{H}_{2n-2t_1+s_1}\left(\frac{y_1'}{w_0}\right)\times \mathrm{H}_{2t_2+m-s_2}\left(\frac{x_2'}{w_0}\right)\mathrm{H}_{2n-2t_2+s_2}\left(\frac{y_2'}{w_0}\right)\times \exp\left(-\frac{{\rho_2'}^2+{\rho_1'}^2}{{w_0}^2}\right)\exp\left[\frac{-(\rho_1'-\rho_2')^2}{2{\delta_{ij}}^2}\right] \tag{6}$$

现在考虑在源平面处，PCELG 光束被一个圆形孔径截断，其半径为 a，则截断的 PCELG 光束在 $z=0$ 平面处的交叉谱密度可以表示为：

$$W_{\mathrm{T},ij}(\rho_1',\rho_2',0) = H(\rho_1')H(\rho_2')\times W_{ij}(\rho_1',\rho_2',0) \tag{7}$$

式中，下标 T 表示的是通光光阑，$H(\rho_1')$为硬边孔径的传输函数，可以展开为复高斯函数的有限项之

和[36-37]：

$$H(\rho_1{}') = \sum_{h=1}^{H} A_h \exp\left(-\frac{B_h \rho_1{}'^2}{a^2}\right) = \begin{cases} 1,(\rho_1{}' \leqslant a) \\ 0,(\rho_1{}' > a) \end{cases} \tag{8}$$

将(6)式、(8)式代入(7)式中，可得到源平面 PCELG 光束通过环形光阑的交叉谱密度矩阵元的解析表达式：

$$W_{\mathrm{T},ij}(\rho_1{}',\rho_2{}',0) = \frac{A_i A_j B_{ij}}{2^{4n+2m} \cdot (n!)^2} \times$$

$$\sum_{h=1}^{H}\sum_{p=1}^{H}\sum_{t_1=0}^{n}\sum_{s_1=0}^{m}\sum_{t_2=0}^{n}\sum_{s_2=0}^{m} A_h A_p{}^* \mathrm{i}^{s_1}(-\mathrm{i})^{s_2} \begin{bmatrix} n \\ t_1 \end{bmatrix} \begin{bmatrix} m \\ s_1 \end{bmatrix} \begin{bmatrix} n \\ t_2 \end{bmatrix} \times$$

$$\begin{bmatrix} m \\ s_2 \end{bmatrix} \exp\left(-\frac{B_h}{\sigma^2}\rho_1{}'^2 - \frac{B_p^*}{\sigma^2}\rho_2{}'^2\right) \mathrm{H}_{2t_1+m-s_1}(X_1{}') \times$$

$$\mathrm{H}_{2n-2t_1+s_1}(Y_1{}')\mathrm{H}_{2t_2+m-s_2}(X_2{}') \times$$

$$\mathrm{H}_{2n-2t_2+s_2}(Y_2{}')\exp\left[-(\rho_2{}'^2 + \rho_1{}'^2)\right] \times$$

$$\exp\left[-\frac{1}{2\delta_{0ij}}(\rho_1{}' - \rho_2{}')^2\right] \tag{9}$$

式中，$\sigma = a/w_0$ 为环形光阑的截断参量，$\delta_{0ij} = \sigma_{ij}/w_0$，$X' = x'/w_0$，$Y' = y'/w_0$。

根据广义的惠更斯-菲涅耳原理[38-39]，在传输距离 z 处，PCELG 光束在湍流大气中交叉谱密度矩阵元为：

$$W_{\mathrm{T},ij}(\rho_1,\rho_2,z,\omega) = \left(\frac{k}{2\pi z}\right)^2 \times$$

$$\iiiint \mathrm{d}^2\rho_1{}'\mathrm{d}^2\rho_2{}' W_{\mathrm{T},ij}(\rho_1{}',\rho_2{}',0,\omega) \times$$

$$\exp\left\{-\frac{\mathrm{i}k}{2z}\left[(\rho_1 - \rho_1{}')^2 - (\rho_2 - \rho_2{}')^2\right]\right\} \times$$

$$\langle \exp[\psi(\rho_1,\rho_1{}',z,\omega) + \psi^*(\rho_2,\rho_2{}',z,\omega)]\rangle_m \tag{10}$$

式中，k 表示波数，$\psi(\rho,\rho',z,\omega)$ 表示大气湍流对球面波影响的随机相位因子。采用 Rytov 二次函数的相位近似来处理湍流，则 $\langle \exp[\psi(\rho_1,\rho_1{}',z,\omega) + \psi^*(\rho_2,\rho_2{}',z,\omega)]\rangle_m$ 可表示为[40]：

$$\langle \exp[\psi(\rho_1,\rho_1{}',z,\omega) + \psi^*(\rho_2,\rho_2{}',z,\omega)]\rangle_m =$$

$$\exp\left\{-\frac{1}{\rho_0{}^2}\left[(\rho_1 - \rho_2)^2 + (\rho_1 - \rho_2)(\rho_1{}' - \rho_2{}') + (\rho_1{}' - \rho_2{}')^2\right]\right\} \tag{11}$$

式中，ρ_0 为源横截面平面坐标。

将(9)式和(11)式代入(10)式得到：

$$W_{\mathrm{T},ij}(\rho_1,\rho_2,z,\omega) = \left(\frac{1}{\pi z}\right)^2 \times$$

$$\iiiint \mathrm{d}^2\rho_1{}'\,\mathrm{d}^2\rho_2{}' W_{\mathrm{T},ij}(\rho_1{}',\rho_2{}',0,\omega) \times$$

$$\exp\left\{-\frac{\mathrm{i}}{z}\left[\rho_1{}'^2 - 2\rho_1 \cdot \rho_1{}' + \rho_1{}^2 - \rho_2{}'^2 + 2\rho_2 \cdot \rho_2{}' - \rho_2{}^2\right]\right\} \times$$

$$\exp\left\{-\frac{1}{\rho_0{}'^2}\left[(\rho_1 - \rho_2)^2 + (\rho_1 - \rho_2)(\rho_1{}' - \rho_2{}') + (\rho_1{}' - \rho_2{}')^2\right]\right\} \tag{12}$$

运用数学公式[41]：

$$\begin{cases} \int \exp[-(x-y)^2]\mathrm{H}_n(\alpha x)\mathrm{d}x = \\ \sqrt{\pi}(1-\alpha^2)^{n/2}\mathrm{H}_n\left[\dfrac{ay}{(1-\alpha^2)^{1/2}}\right] \\ (a+b)^n = \displaystyle\sum_{k}^{n} \begin{bmatrix} n \\ k \end{bmatrix} a^k b^{n-k} \\ \mathrm{H}_n(x+y) = \dfrac{1}{2^{n/2}} \displaystyle\sum_{k=0}^{n} \mathrm{H}_k(\sqrt{2}x)\mathrm{H}_{n-k}(\sqrt{2}y) \\ \int x^n \exp(-px^2 + 2qx)\mathrm{d}x = n!\sqrt{\dfrac{\pi}{p}}\left(\dfrac{q}{p}\right)^n \times \\ \exp\left(\dfrac{q^2}{p}\right) \displaystyle\sum_{k=0}^{[n/2]} \dfrac{1}{(n-2k)!k!}\left(\dfrac{p}{4q^2}\right)^k \end{cases} \tag{13}$$

经过一系列复杂的数学运算，可得交叉谱密度矩阵最后结果：

$$W = \frac{A_i A_j B_{ij}}{2^{4n+2m}(n!)^2 z^2} \sum_{h=1}^{H}\sum_{p=1}^{H}\sum_{t_1=0}^{n}\sum_{s_1=0}^{m}\sum_{t_2=0}^{n}\sum_{s_2=0}^{m}\sum_{k_1=0}^{2t_1+m-s_1}\sum_{k_2=0}^{2n-2t_1+s_1}\sum_{g_1=0}^{\left[\frac{2t_2+m-s_2}{2}\right]}\sum_{g_2=0}^{\left[\frac{2n-2t_2+s_2}{2}\right]}\sum_{h_1=0}^{\left[\frac{k_1}{2}\right]}\sum_{h_2=0}^{\left[\frac{k_2}{2}\right]} \frac{1}{p_{1ij}p_{2ij}} \times$$

$$A_h A_p{}^* \begin{bmatrix} n \\ t_1 \end{bmatrix} \begin{bmatrix} m \\ s_1 \end{bmatrix} \begin{bmatrix} n \\ t_2 \end{bmatrix} \begin{bmatrix} m \\ s_2 \end{bmatrix} \begin{bmatrix} 2t_1 + m - s_1 \\ k_1 \end{bmatrix} \begin{bmatrix} 2n - 2t_1 + s_1 \\ k_2 \end{bmatrix} (-1)^{s_2+g_1+g_2+h_1+h_2} \times$$

$$\frac{(2t_2 + m - s_2)!}{g_1!(2t_2 + m - s_2 - 2g_1)!} \frac{(2n - 2t_2 + s_2)!}{g_2!(2n - 2t_2 + s_2 - 2g_2)!} \frac{k_1!}{h_1!(k_1 - 2h_1)!} \frac{k_2!}{h_2!(k_2 - 2h_2)!} \times$$

$$\mathrm{i}^{s_1+s_2-(m+2n-2g_1-2g_2+k_1+k_2-2h_1-2h_2)}(p_{1ij} - 1)^{(m+2n-k_1-k_2+2h_1+2h_2)/2} 2^{(k_1+k_2-2h_1-2h_2-m-2n)/2} \times$$

$$2^{2h_1-2h_2-2g_2+2g_1}\left(\frac{1}{{\rho_i}^2}\right)^{k_1+k_2-2h_1-2h_2}{p_{1ij}}^{-(m+2n+k_1+k_2-2h_1-2h_2)/2}{p_{2ij}}^{-(m+2n-2g_1-2g_2+k_1+k_2-2h_1-2h_2)/2}\times$$

$$\mathrm{H}_{2t_1+m-s_1-k_1}\left(\frac{\sqrt{2}q_{1x}}{\sqrt{p_{1ij}}(p_{1ij}-1)^{1/2}}\right)\mathrm{H}_{2n-2t_1+s_1-k_2}\left(\frac{\sqrt{2}q_{1y}}{\sqrt{p_{1ij}}(p_{1ij}-1)^{1/2}}\right)\times$$

$$\mathrm{H}_{2t_2+m-s_2-2g_1+k_1-2h_1}\left(\mathrm{i}\frac{q_{2ij}}{\sqrt{p_{2ij}}}\right)\mathrm{H}_{2n-2t_2+s_2-2g_2+k_2-2h_2}\left(\mathrm{i}\frac{q_{2yij}}{\sqrt{p_{2ij}}}\right)\exp\left[\frac{{q_{1x}}^2}{p_{1ij}}+\frac{{q_{2ij}}^2}{p_{2ij}}\right]\times$$

$$\exp\left[\frac{{q_{1y}}^2}{p_{1ij}}+\frac{{q_{2yij}}^2}{p_{2ij}}\right]\exp\left[-\frac{\mathrm{i}}{2}({\rho_1}^2-{\rho_2}^2)-\frac{1}{{\rho_0}'^2}(\rho_1-\rho_2)^2\right]\tag{14}$$

其中：

$$\frac{1}{\rho^2}=\frac{1}{2\delta_{0ij}}+\frac{1}{{\rho_0}'^2},p_{1ij}=\frac{B_h}{\sigma^2}+1+\frac{1}{2\delta_{0ij}}+\frac{1}{{\rho_0}'^2}+\frac{\mathrm{i}}{z},q_{1x}=\frac{\mathrm{i}}{z}x_1-\frac{1}{2{\rho_0}'^2}(x_1-x_2),$$

$$p_{2ij}=\frac{B_p^*}{\sigma^2}+1+\frac{1}{2\delta_{0ij}}-\frac{\mathrm{i}}{z}+\frac{1}{{\rho_0}'^2}-\frac{1}{p_{1ij}\rho^4},q_{2ij}=\frac{1}{2{\rho_0}'^2}(x_1-x_2)+\frac{q_{1x}}{p_{1ij}\rho^2}-\frac{\mathrm{i}}{z}x_2,$$

$$p_{1ij}=\frac{B_h}{\sigma^2}+1+\frac{1}{2\delta_{0ij}}+\frac{1}{{\rho_0}'^2}+\frac{\mathrm{i}}{z},q_{1y}=\frac{\mathrm{i}}{z}y_1-\frac{1}{2{\rho_0}'^2}(y_1-y_2),$$

$$q_{2yij}=-\frac{\mathrm{i}}{z}y_2+\frac{1}{2{\rho_0}'^2}(y_1-y_2)+\frac{q_{1y}}{p_{1ij}\rho^2}\tag{15}$$

在任意 $z>0$ 平面内,PCELG 光束的偏振度表示为[6,9]：

$$P(\rho)=\sqrt{1-\frac{4\det W(\rho,\rho)}{[\mathrm{tr}W(\rho,\rho)]^2}}=\frac{[(W_{xx}-W_{yy})^2+4W_{xy}W_{yx}]^{1/2}}{W_{xx}+W_{yy}}\tag{16}$$

式中,det 和 tr 分别表示交叉谱密度矩阵对应行列式的值和迹。

2　数值计算和分析

根据前面所得到的解析表达式(13)式、(14)式和(15)式,选取不同参量进行相应的数值计算。图 1a 和图 1b 分别表示当传输距离 $z=1\mathrm{km}$ 和 $z=3\mathrm{km}$ 时,阶数 $m=n=1$,相干长度 $\delta_{xx}=0.01\mathrm{m}$, $\delta_{yy}=0.05\mathrm{m}$ 的 PCELG 光束通过截断参量等于 1 的光阑后,在自由空间和不同强度的湍流大气中传输时,接收面横轴方向上偏振度的变化情况。从图 1 可以看出,无论是在湍流大气还是在自由空间中传输,该光束横轴方向上的偏振度都出现剧烈的振荡现象。这主要是由于光阑的衍射效应,使得光束偏振行为复杂化。然而,在离轴较远点处,偏振度总是趋向于 1。且随着传输距离的增加,其横向偏振度趋向于 1 时所对应的离轴距离也增加。

图 2a 和图 2b 分别表示传输距离 $z=0\mathrm{km}\sim1\mathrm{km}$ 和 $z=0\mathrm{km}\sim6\mathrm{km}$ 时,PCELG 光束通过截断参量等于 1 的光阑后,在不同强度的湍流大气中传输时轴上偏振度的变化情况。由图 2a 可以看出,当 PCELG 光束在 $z=0\mathrm{m}\sim100\mathrm{m}$ 传输时,随着传输距离的增加,初始偏振度由 $P_0=0.6$ 开始,先减小后迅速增加直至趋向于 1。当传输距离继续增加时($z>100\mathrm{m}$),偏振度逐渐减小,且湍流强度越强,偏振度下降的越快。由图 2b 可以看出,随着传输距离的增加($z\geqslant1\mathrm{km}$),不同湍流强度所对应的偏振度依次减小为

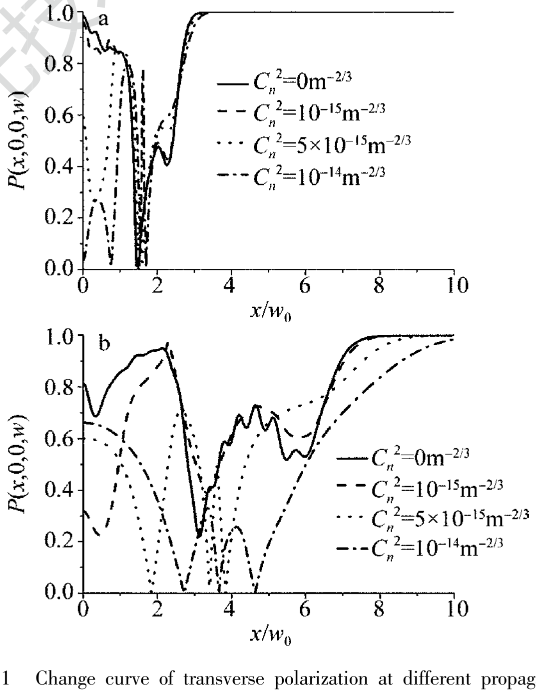

Fig. 1　Change curve of transverse polarization at different propagation distances z while $\sigma=1$, $m=n=1$, $\delta_{xx}=0.01\mathrm{m}$, $\delta_{yy}=0.05\mathrm{m}$
a—$z=1\mathrm{km}$　b—$z=3\mathrm{km}$

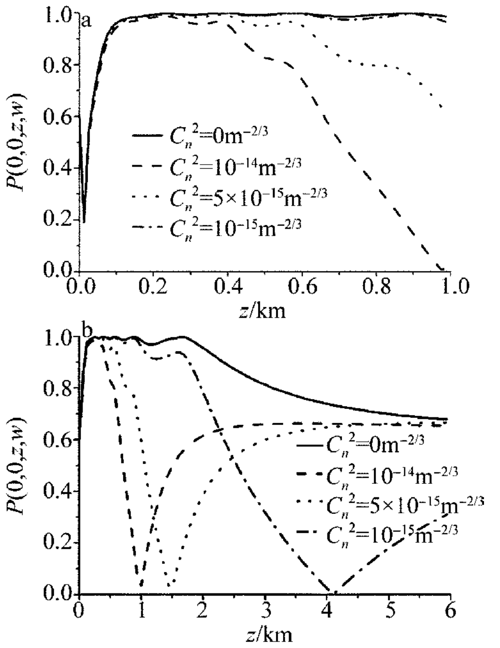

Fig. 2 Change curve of polarization in on-axis observation point at propagation distance z for different C_n^2 while $\sigma=1$, $m=2$, $n=1$, $\delta_{xx}=0.01\mathrm{m}$, $\delta_{yy}=0.05\mathrm{m}$

a—$z=0\mathrm{km}\sim1\mathrm{km}$ b—$z=0\mathrm{km}\sim6\mathrm{km}$

0(自由空间中除外)，然后逐渐增加。当经过足够长的距离后，光束在不同强度的湍流大气中传输时的偏振度都趋近于初始偏振度 $P_0=0.6$。

图 3a 和图 3b 分别表示 PCELG 光束通过不同截断参量的光阑后，在自由空间和湍流大气中传输时轴上偏振度随传输距离 z 的变化情况。从图 3 可以看出，在自由空间中，通过截断参量等于 0.5 的光阑，PCELG 光束在很短的传输距离内偏振度减小得较快，然后又经过较短的传输距离后趋向于一个定值，而截断参量变大时，光束偏振度振荡行为加剧，经过足够长的距离后，分别趋向于各自定值。对于通过不同截断参量的光阑，光束在湍流大气中传输足够长的距离后，偏振度都趋向于初始偏振度 $P_0=0.6$。

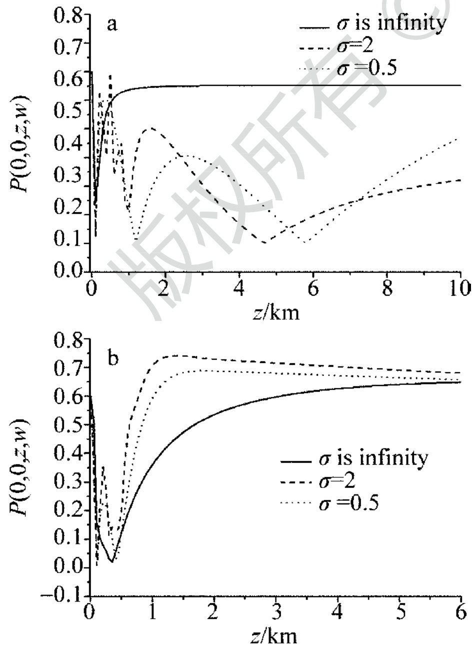

Fig. 3 Change curve of polarization in on-axis observation point at propagation distance z for different σ while $m=n=1$, $w_0=0.02\mathrm{m}$, $\delta_{xx}=0.01\mathrm{m}$, $\delta_{yy}=0.05\mathrm{m}$

a—$C_n^2=0\mathrm{m}^{-2/3}$ b—$C_n^2=10^{-14}\mathrm{m}^{-2/3}$

图 4a 和图 4b 分别表示不同源偏振度的光束，通过截断参量等于 0.5 的环形光阑后，在自由空间和湍流大气中传输时，轴上偏振度随着传输距离 z 的变化情况。由图 4a 可以看出，在自由空间中传输时，经过足够长的传输距离后，光束的偏振度趋近于各自极限值，且不等于各自的源偏振度。完全非偏振光($P_0=0$)变为部分偏振光($P_0\neq0$)。然而，在湍流大气中传输时，经过足够长的传输距离后，其偏振度趋向于各自的源偏振度。

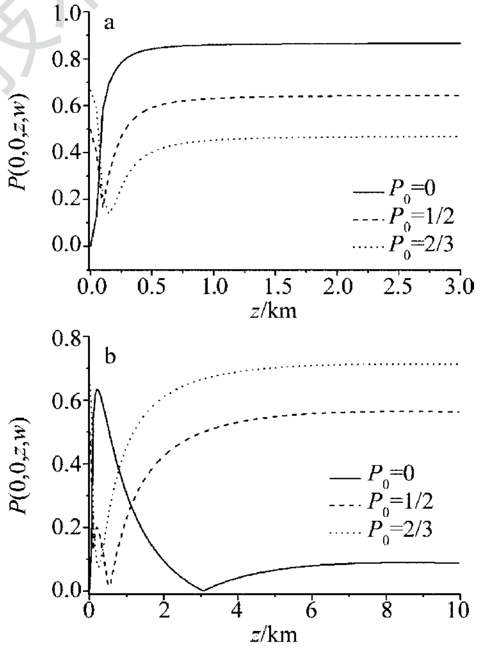

Fig. 4 Change curve of polarization in on-axis observation point at propagation distance z while $m=n=1$, $\sigma=0.5$, $\delta_{xx}=0.01\mathrm{m}$, $\delta_{yy}=0.05\mathrm{m}$

a—$C_n^2=0\mathrm{m}^{-2/3}$ b—$C_n^2=10^{-14}\mathrm{m}^{-2/3}$

3 结 论

基于偏振性和相干性统一理论，根据拓展的惠更斯-菲涅耳原理，理论推导了 PCELG 光束的交叉谱密度和偏振度的解析表达式，并对其偏振度进行了数值计算。结果表明：PCELG 光束的偏振度受光阑截断参量、大气折射率结构常数及源偏振度的影

响。PCELG 光束通过环形光阑后，由于光阑的衍射效应，使得该光束横轴方向上的偏振行为复杂化。当光束在湍流大气和自由空间中传输一定的距离后，其横向偏振度都是趋向于 1，且随着传输距离的增加，其横向偏振度趋向于 1 时所对应的离轴距离也增加。当 PCELG 光束通过光阑后，在自由空间中传输时，经过足够长的传输距离，偏振度趋向于各自不同的极限值，而在湍流大气中传输时，偏振度则是趋向于初始偏振度；光阑的截断参量越大，轴上偏振度的振荡行为越剧烈。所得结果对光束在大气中的传输及其应用具有实际意义。

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