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Compton 散射对未磁化等离子体调制不稳定性的影响

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摘要: 为了研究 Compton 散射对未磁化等离子体调制不稳定性的影响, 采用多光子非线性 Compton 散射模型, 对 Compton 散射下的未磁化等离子体调制不稳定性进行了理论分析和实验验证, 取得了散射对调制不稳定性的时间增长率、非线性色散和控制影响的重要数据, 提出了将入射光和 Compton 散射光作为形成调制不稳定性的新机制。结果表明, Compton 散射使未磁化等离子体的自调制不稳定性的最大时间增长率和激光自聚焦较散射前减小, 等离子体截面附近处调制不稳定性的时间增长率较散射前增大。

关键词: 激光物理; 未磁化等离子体; 耦合; 左旋椭圆偏振光; 调制不稳定性; 多光子非线性 Compton 散射
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Influence of Compton scattering on the modulation instability in un-magnetized plasma

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Abstract: In order to study effect of Compton scattering on the modulation instability in the un-magnetized plasma, the modulation instability of the un-magnetized plasma under Compton scattering was analyzed and simulated based on the model of the multi-photon nonlinear Compton scattering. Some important data about effect of Compton scattering on the time rise rate of the modulation instability, the nonlinear dispersion and control were obtained, and a new mechanism of the modulation instability formed by the incident light and the Compton scattering was put forward. The result shows that the maximum time rise rate of the modulation instability and laser self-focusing in the un-magnetized laser-plasma becomes smaller than before the Compton scattering, and the time rise rate of the modulation instability in the vicinity of the laser-plasma surface is increased.

Key words: laser physics; un-magnetized plasma; coupling; left-hand elliptically polarized light; modulation instability; multi-photon nonlinear Compton scattering

引 言

高功率短激光脉冲与靶材作用, 使聚集在小区域的部分激光能量被靶材吸收来不及热传导, 导致靶材加热和电离产生等离子体。等离子体中, 只有频率大于等离子体频率的激光才能传播, 即存在反射激光临界面, 激光在临界面附近形成的横等离激元与离声波调制形成的包络出现调制不稳定性[1] (modulation instability, MI)。由于 MI 被广泛用于惯性约束聚变[2]、粒子加速[3]等, 因此倍受人们关注[4-6]。LIU 等人[7-8]给出了等离子体中自生磁场产生的机制, 发现等离子体界面附近有很强的自生磁场产生。JHA 等人[9]指出, 自生磁场和外磁场使等离子体 MI 增强, 激光利用率减低, 产生的超热电子对内爆压缩过程起破坏作用[10]。SPRAGLE 等人[11]给出了等离子体调制增长率空间分布。JHA 等人[12]发现, 激光功率超过 10^{18} W/cm^2 时, 电子有很高的颤动能量, 须考虑相对论效应。CHEN 等人[13-14]指出, 磁化等离子体的自调制不稳定性(self-modulation instability, SMI)的极大增长率比非磁化情况明显减小, 等离子体界面附近 MI 的时间增长率显著增大。最近, YAO 等人[15]指出, Compton 散射使磁化等离子体中和截面处的 SMI 最大时间增长率比散射前减小和增大。应指出的是, 以上研究未考虑 Compton 散射对非磁化等离子体的 MI 的影响。实验表明, 光强为 10^{16} W/cm^2 数量级以上时, 非线性 Compton 效应开始显现[16]。而且, 随着脉冲啁

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啾放大技术的重大突破，目前在实验室已可实现这种散射。可见，研究 Compton 散射对非磁化等离子体 MI 影响是非常必要的。

1　Compton 散射对色散的影响

若等离子体中发生多光子非线性 Compton 散射(以下简称散射)，则散射光子频率为[17]：

$$\omega_s = \frac{N\omega(1+\beta\cos\theta)(1-\beta_f\cos\theta_1')}{\xi^2+\xi N\hbar\omega(1+\beta\cos\theta)\dfrac{1-\cos\theta'}{m_0c^2}} \qquad (1)$$

式中，$\xi=\frac{|\gamma-\gamma_f|}{\gamma-1}$ 是量度散射非弹性参量；$\gamma_{(f)}=\left[1-\frac{v_{(f)}}{c}^2\right]^{-1/2}=(1-\beta_{(f)}^2)^{-1/2}$ 为电子散射前的洛伦兹因子，$v_{(f)}$ 为速率；θ_1' 和 θ' 为电子静止系(S'系)中电子与散射光子运动方向夹角和光子散射角；$N,m_0,\omega,k,c,h=2\pi\hbar$ 和 θ 分别为与电子同时作用的光子数、电子静质量、入射光频率、波数、真空中的光速、普朗克常数以及电子和光子散射前运动方向夹角。若取散射光与入射光形成的耦合光频率 $\omega_c=\omega_s-\omega$，则有：

$$\omega_c = \omega\left[\frac{N(1+\beta\cos\theta)(1-\beta_f\cos\theta_1')}{\xi^2+\xi N\hbar\omega(1+\beta\cos\theta)\dfrac{1-\cos\theta'}{m_0c^2}}-1\right] \qquad (2)$$

设入射光和散射光沿 z 方向的磁场分别为 $\boldsymbol{B}$ 和 $\Delta\boldsymbol{B}$，这两束光形成的耦合左旋椭圆偏振光的电场矢量为：

$$\boldsymbol{E}_c = \boldsymbol{E}_c(\boldsymbol{r},t)\exp[i(k_cz-\omega_ct)] = (E_x\boldsymbol{e}_x-iE_y\boldsymbol{e}_y)\exp[i(k_cz-\omega_ct)] \qquad (3)$$

式中，$\boldsymbol{E}_c(\boldsymbol{r},t)=\boldsymbol{E}(\boldsymbol{r},t)+\Delta\boldsymbol{E}(\boldsymbol{r},t)$，$\boldsymbol{k}_c=\boldsymbol{k}+\Delta\boldsymbol{k}$ 为耦合波函数和波矢；$\boldsymbol{E}$ 和 $\boldsymbol{k}$，$\Delta\boldsymbol{E}$ 和 $\Delta\boldsymbol{k}$，$\boldsymbol{r}$，t，$\boldsymbol{e}_x$ 和 $\boldsymbol{e}_y$ 分别为入射光电场、波矢及其增量，位置矢量，时间，x 和 y 方向单位矢量。电子运动方程为：

$$\frac{d(\gamma m_0\boldsymbol{v})}{dt}+\frac{d(m_0\boldsymbol{v}+\gamma m_0\Delta\boldsymbol{v})}{dt} = e\left(\boldsymbol{E}+\frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)+e\left(\Delta\boldsymbol{E}+\frac{\Delta\boldsymbol{v}\times\boldsymbol{B}+\boldsymbol{v}\times\Delta\boldsymbol{B}}{c}\right) \qquad (4)$$

式中，e，$\boldsymbol{v}_c=\boldsymbol{v}+\Delta\boldsymbol{v}$，$\boldsymbol{v}$ 和 $\Delta\boldsymbol{v}$ 分别为电子电量、耦合速度、散射前速度及其增量；$\gamma_c=\gamma+\Delta\gamma$，$\Delta\gamma$ 为 γ 增量，$\Delta\gamma=\left[1-\frac{\Delta\boldsymbol{v}^2}{c}\right]^{-1/2}\approx 1$；等号两端第 2 项为修正项。(4)式忽略了 Δ^2 项(以下均如此)。

设 S'系相对于实验室系(S系)以速率 v 沿 z 向运动，$v=n_cc$，$n_c=n+\Delta n$，n 和 Δn 分别为耦合折射率、散射前折射率及其增量；$\frac{\partial\boldsymbol{E}_c'}{\partial t'}=4\pi\tilde{N}'|e|\boldsymbol{v}_c$，$\tilde{N}'$是电子数密度。$S'$系中，(4)式可写为：

$$\left(-m\frac{d\boldsymbol{v}'}{dt'}-\frac{|e|\boldsymbol{v}'\times\boldsymbol{B}'}{c^2}\right)-\left[m\frac{d\Delta\boldsymbol{v}'}{dt'}-\frac{|e|(\Delta\boldsymbol{v}'\times\boldsymbol{B}'+\boldsymbol{v}'\times\Delta\boldsymbol{B}')}{c^2}\right]=\frac{|e|}{c}\boldsymbol{E}'+\frac{|e|}{c}\Delta\boldsymbol{E}' \qquad (5)$$

式中，$\boldsymbol{v}'=\frac{\gamma'\boldsymbol{v}}{c}$ 和 $\Delta\boldsymbol{v}'=\frac{\boldsymbol{v}+\gamma'\Delta\boldsymbol{v}}{c}$ 分别为散射前约化速度及其增量。电子运动方程为：

$$\left(\frac{d^2\boldsymbol{v}'}{dt'^2}+\frac{\omega_p'^2\boldsymbol{v}'}{\gamma'}+\frac{|e|}{m_0c^2}\frac{d}{dt'}\boldsymbol{v}'\times\boldsymbol{B}'\right)-\left[\frac{d^2\Delta\boldsymbol{v}'}{dt'^2}+\frac{\omega_p'^2\Delta\boldsymbol{v}'}{\gamma'}+\frac{|e|}{m_0c^2}\frac{d(\Delta\boldsymbol{v}'\times\boldsymbol{B}'+\boldsymbol{v}'\times\Delta\boldsymbol{B}')}{dt'}\right]=0 \qquad (6)$$

式中，$\omega_p'=\left(\frac{4\pi\tilde{N}'e^2}{m_0}\right)^{1/2}$ 为等离子体频率；等号左端第 2 项为修正项。设(6)式形式解为 $\boldsymbol{v}_c'=\boldsymbol{v}_c+\Delta\boldsymbol{v}'=(\boldsymbol{v}'+\Delta\boldsymbol{v}')_\perp e^{-i\omega_c't'}+(v_{z'}'+\Delta v_{z'}')\boldsymbol{e}_{z'}$，$(\boldsymbol{v}'+\Delta\boldsymbol{v}')_\perp$ 是 $(v_{z'}'+\Delta v_{z'}')$ 为垂直和平行 z' 方向分量，$e^{-i\omega_c't'}$ 为缓变函数。忽略分母中的 Δ 项(以下均如此)，得色散方程为：

$$\omega_c'^2\approx\left(\frac{\omega_p'^2}{\gamma'}-\frac{|e|\omega'B'}{m_0c\gamma'}\right)-\left(|e|\frac{\Delta\omega'B'+\omega'\Delta B'}{m_0c\gamma'}\right) \qquad (7)$$

(7)式为修正色散方程，等号右端第 2 个括号中的项为修正项。由(5)式，得：

$$\boldsymbol{E}_c'=(\boldsymbol{E}'+\Delta\boldsymbol{E}')_\perp\exp(i\omega_c't') \qquad (8)$$

式中，$(\boldsymbol{E}'+\Delta\boldsymbol{E}')_\perp\approx\frac{m_0c\omega_c'\boldsymbol{v}_\perp'}{|e|}-\frac{(\boldsymbol{v}'\times\boldsymbol{B}')_\perp}{\gamma'}-\left[\frac{m_0c\omega_c'\boldsymbol{v}_\perp'}{|e|}+\frac{\Delta\boldsymbol{v}'\times\boldsymbol{B}'+\boldsymbol{v}'\times\Delta\boldsymbol{B}'}{\gamma'}\right]$。由洛伦兹变换及 $\gamma_c^2=1+v_{c\perp}^2+v_{cz}^2$，$S$ 系中色散方程为：

$$\omega_c^2=k_c^2c^2+\frac{(1-v_c^2/c^2)^{1/2}}{\sqrt{1+v_{c\perp}^2}}\left(\frac{\omega_c\omega_p^2}{\omega_c+\omega_h}\right) \qquad (9)$$

式中，$\omega_h=\omega_h+\Delta\omega_h=|e|\frac{B}{m_0c}+|e|\frac{\Delta B}{m_0c}$ 为电子回旋共振频率，设 $\omega_h\ll\omega_c$；$\omega_h=\frac{|e|B}{m_0c}$ 和 $\Delta\omega_h=\frac{|e|\Delta B}{m_0c}$ 为散射前电子回旋共振频率及其增量。S 系中，电子约化速度分量 $v_{cz}=v_z-\Delta v_z\approx\frac{\gamma v}{c}+\frac{v-\gamma\Delta v}{c}$，约等号两端第 2 项为 v_{cz} 增量。设 $v_c\ll c$，结合(8)式，得 S 系中色散关系为：

$$\omega_c^2=c^2(k^2+2k\Delta k)+$$

$$\left[1+\frac{e^2 E_\perp{}^2+2E_\perp \Delta E_\perp}{m_0{}^2c^2(\omega_c{}^2+\omega_h{}^2)}\right]^{1/2}\left(\frac{\omega_c\omega_p{}^2}{\omega_c+\omega_h}\right) \quad (10)$$

等号右端第 2 项表明，散射使相对论和非线性效应增强。将(8)式从 S' 变到 S 系，$E_c=\psi_c\exp[-i(\omega_c t-k_c z)]$，横向包络 $\psi_{c\perp}\exp\{-i[(\omega_c-\omega)t+i(k_c-k)z]\}=(E+\Delta E)_\perp\exp[-i(\Delta\omega t-\Delta kz)]$ 是受非线性方程控制的函数。

2 对控制方程的修正

取横向包络 $\psi_c=a_c e^{i\theta_c}$，$\psi_c=\psi+\Delta\psi$，$a_c=a+\Delta a$，$\theta_c=\theta+\Delta\theta$；$a_c$ 和 θ_c 是耦合振幅和相位，ψ，a，θ 及 $\Delta\psi$，Δa，$\Delta\theta$ 分别是散射前横向包络、振幅、相位及其增量；$\theta_c=(k+\Delta k)z-(\omega+\Delta\omega)t+(\varphi+\Delta\varphi)$，$\varphi_c=\varphi+\Delta\varphi$，$\varphi$ 和 $\Delta\varphi$ 分别是耦合函数、散射前函数及其增量。将 ω_c 在 k 附近展开，得：

$$\omega_c\approx\left\{\omega+\left(\frac{\partial\omega}{\partial k}\right)_k\left[\varphi_z+\frac{(\nabla_\perp\varphi)^2}{2k}\right]+\frac{1}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)\right\}_k+\left\{\Delta\omega+\left(\frac{\partial\Delta\omega}{\partial k}\right)_k\left[\varphi_z+\frac{(\nabla_\perp\varphi)^2}{2k}\right]+\left(\frac{\partial\omega}{\partial k}\right)_k\left(\Delta\varphi_z+\frac{\nabla_\perp{}^2\Delta\varphi}{k}\right)+\frac{1}{2}\left(\frac{\partial^2\Delta\omega}{\partial k^2}\right)_k\varphi_z^2+\left(\frac{\partial^2\omega}{\partial k^2}\right)_k\varphi_z\Delta\varphi_z\right\} \quad (11)$$

式中，∇ 为哈密顿；φ_z 和 $\Delta\varphi_z$ 为垂直 z 分量及其增量。用非线性色散 $\tilde{\omega}(k_c,a_c)$ 代替 $\omega(k_c)$，$\tilde{\omega}_c=\omega_c+\eta_c|a_c|^2$。结合(10)式，有 $\eta_c=e^2\omega_c(\omega_h-\omega_c)/[2m_0{}^2c^2(\omega_c{}^2+\omega_h{}^2)\times(2\omega_c{}^3-\omega_p{}^2\omega_h)]$。将 E_c 展为傅氏积分，由 $\tilde{\omega}_c-\omega_c\to\frac{i\partial}{\partial t}$，$(\nabla_\perp\varphi_c)^2\to(\nabla\times\nabla\times)_\perp-(\nabla\nabla\cdot)_z$，则有：

$$\left\{i\left(\frac{\partial\psi}{\partial t}+v_g\frac{\partial\psi}{\partial z}\right)-\frac{a_g}{2}[(\nabla\times\nabla\times)_z-(\nabla\nabla\cdot)_z]\psi-\eta|\psi|^2\psi-\frac{a_g}{2}[(\nabla\times\nabla\times)_\perp-(\nabla\nabla\cdot)_\perp]\psi\right\}+\left\{i\left(\frac{\partial\Delta\psi}{\partial t}+\Delta v_g\frac{\partial\psi}{\partial z}+v_g\frac{\partial\Delta\psi}{\partial z}\right)-\frac{a_g}{2}[(\nabla\times\nabla\times)_z-(\nabla\nabla\cdot)_z]\Delta\psi-\frac{\Delta a_g}{2}[(\nabla\times\nabla\times)_z-(\nabla\nabla\cdot)_z]\Delta\psi-\frac{v_g}{2}[(\nabla\times\nabla\times)_\perp-(\nabla\nabla\cdot)_\perp]\Delta\psi-\Delta\eta|\psi|^2\psi-3\eta|\psi|^2\Delta\psi+\frac{\Delta v_g}{2}[(\nabla\times\nabla\times)_\perp-(\nabla\nabla\cdot)_\perp]\psi\right\}=0 \quad (12)$$

式中，$v_{c,g}=v_g+\Delta v_g$ 和 $a_{c,g}=a_g+\Delta a_g$，v_g 和 a_g，Δv_g 和 Δa_g 分别为耦合群速和群加速度、散射前群速和群加速度及其增量。第 1 个花括号中的第 4 项和第 2 个花括号中的第 6 项～第 10 项分别为密度扰动和散射对调制的贡献。

3 MI 分析

引入参量 $\tilde{t}=k_c{}^2a_{c,g}t$，$\tilde{x}=\sqrt{2}\left(\frac{k_c a_{c,g}}{v_{c,g}}\right)^{1/2}x$；$\tilde{y}=\sqrt{2}\left(\frac{k_c a_{c,g}}{v_{c,g}}\right)^{1/2}y$；$\tilde{\psi}_c=\frac{|e|\psi_c}{m_0 c\omega_c}$；$\tilde{z}=\sqrt{2}k_c z$，则(12)式为：

$$\left(\frac{i\partial}{\partial\tilde{t}}+i\eta_1\frac{\partial}{\partial\tilde{z}}+\tilde{\nabla}^2+\eta_2\tilde{N}\right)\tilde{\psi}+\left[\left(\frac{i\partial}{\partial\tilde{t}}+\tilde{\nabla}^2+i\eta_1\frac{\partial}{\partial\tilde{z}}+\eta_2\tilde{N}\right)\Delta\tilde{\psi}+\left(i\Delta\eta_1\frac{\partial}{\partial\tilde{z}}+\eta_2\Delta\tilde{N}+\Delta\eta_2\tilde{N}\right)\psi\right]=0 \quad (13)$$

$$\tilde{\nabla}^2\tilde{N}+\tilde{\nabla}^2\Delta\tilde{N}=-\tilde{\nabla}^2|\tilde{\psi}|^2-\tilde{\nabla}^2|2\tilde{\psi}\Delta\tilde{\psi}| \quad (14)$$

式中，$\eta_1\approx\frac{\sqrt{2}v_g}{ka_g}$，$\Delta\eta_1\approx\frac{\sqrt{2}\Delta v_g}{ka_g}$；$\eta_2\approx\frac{\eta m_0{}^2c^2\omega^2}{e^2k^2a_g{}^2}$，$\Delta\eta_2\approx\frac{\Delta\eta m_0{}^2c^2\omega^2}{e^2k^2a_g{}^2}$；$\tilde{N}_c=\tilde{N}+\Delta\tilde{N}$ 为电子耦合数密度，$\tilde{N}$ 和 $\Delta\tilde{N}$ 是散射前电子数密度及其增量；$\tilde{\psi}_c=\tilde{\psi}+\Delta\tilde{\psi}$。(13)式等号左端第 2 项及(14)式等号两端第 2 项为修正项。若波的形式解为：

$$\tilde{\psi}_{c1}=\tilde{\psi}_{c,m}\exp[i(\tilde{k}_c\tilde{z}-\tilde{\Omega}_c\tilde{t})] \quad (15)$$

式中，$\tilde{\Omega}_c=\tilde{\Omega}+\Delta\tilde{\Omega}\approx(\eta_1\tilde{K}+\tilde{K}^2)+(\Delta\eta_1\tilde{K}+\eta_1\Delta\tilde{K}+2\tilde{K}\Delta\tilde{K})$，其中 $\Delta\tilde{\Omega}=\Delta\eta_1\tilde{K}+\eta_1\Delta\tilde{K}+2\tilde{K}\Delta\tilde{K}$ 为修正项；$\tilde{K}_c\cdot\tilde{\psi}_{c,m}=0$；$\tilde{K}_c=\tilde{K}_c e_{\tilde{z}}$，$\tilde{K}_c=\tilde{K}+\Delta\tilde{K}$；$\tilde{\psi}_{c,m}=\tilde{\psi}_m+\Delta\tilde{\psi}_m$，$\tilde{\psi}_m$ 和 $\Delta\tilde{\psi}_m$，$\tilde{K}$ 和 $\tilde{\Omega}$，$e_{\tilde{z}}$ 分别是平面波振幅及其增量、波数和频率、$\tilde{z}$ 向单位矢量。若(13)式和(14)式扰动是微扰，振幅被放大，则平面波具有李雅普洛夫意义的 MI。将(13)式和(14)式线性化，有：

$$[i(\Delta\tilde{\psi})_{\tilde{t}}+i\eta_1(\Delta\tilde{\psi})_{\tilde{z}}+\nabla^2\Delta\tilde{\psi}+\eta_2\tilde{N}_2\tilde{\psi}_{c1}]+(\Delta\eta_2\tilde{N}_2\tilde{\psi}_{c1}+\eta_2\Delta\tilde{N}_2\tilde{\psi}_{c1}+\eta_2\tilde{N}_2\Delta\tilde{\psi}_{c1})=0 \quad (16)$$

$$\tilde{\nabla}^2\tilde{N}_2+\tilde{\nabla}^2\Delta\tilde{N}_2=$$

$$-\tilde{\nabla}^2\left[\tilde{\boldsymbol{\psi}}_{c1}\cdot\Delta\tilde{\boldsymbol{\psi}}^*+\tilde{\boldsymbol{\psi}}_{c1}^*\cdot\tilde{\boldsymbol{\psi}}\right]-\tilde{\nabla}^2\left|2\tilde{\boldsymbol{\psi}}_{c1}\Delta\tilde{\boldsymbol{\psi}}\right| \quad (17)$$

考虑如下形式的扰动:

$$\Delta\tilde{\boldsymbol{\psi}}=\tilde{\boldsymbol{\psi}}_{c2}\{\exp[i(\tilde{\boldsymbol{K}}_{c,p}\cdot\tilde{\boldsymbol{r}}-\tilde{\Omega}_{c,p}\tilde{t})]+\exp[-i(\tilde{\boldsymbol{K}}_{c,p}\cdot\tilde{\boldsymbol{r}}-\tilde{\Omega}_{c,p}\tilde{t})]\}\exp[i(\tilde{K}_c\tilde{r}-\tilde{\Omega}\tilde{t})] \quad (18)$$

$$\tilde{N}_2=\frac{\tilde{N}}{2}\{\exp[i((\tilde{K}_{c,p}\cdot\tilde{r}-\tilde{\Omega}_{c,p}\tilde{t})]+\exp[-i(\tilde{K}_{c,p}\cdot\tilde{r}-\tilde{\Omega}_{c,p}\tilde{t})]\} \quad (19)$$

式中,$\tilde{\psi}_{c2}$和$\tilde{\psi}_{c2}^+$及$\tilde{\boldsymbol{K}}_{c,p}$和$\tilde{\Omega}_{c,p}$是横向扰动及波矢和频率;$\tilde{\boldsymbol{\psi}}_{c2}=\tilde{\psi}_{c2}\tilde{e}_{c2}$;$\tilde{\boldsymbol{\psi}}_{c2}^+=\tilde{\psi}_{c2}\tilde{\boldsymbol{e}}_{c2}^+$;$\tilde{e}_{c2}\perp\tilde{\boldsymbol{K}}_{c,p+}$;$\tilde{\boldsymbol{e}}_{c2}^+\perp\tilde{\boldsymbol{K}}_{c,p-}$;$\tilde{\boldsymbol{K}}_{c,p+}=\tilde{\boldsymbol{K}}_p+\tilde{\boldsymbol{K}}_c$;$\tilde{\boldsymbol{K}}_{c,p-}=\tilde{\boldsymbol{K}}_p-\tilde{\boldsymbol{K}}_c$;$\tilde{\Omega}_{c,p+}=\tilde{\Omega}+\tilde{\Omega}_c$;$\tilde{\Omega}_{c,p-}=\tilde{\Omega}_p-\tilde{\Omega}_c$;$\tilde{\boldsymbol{K}}_p$和$\tilde{\Omega}_p$为散射前扰动波矢量和频率;$\tilde{e}_{c2}$和$\tilde{e}_{c2}^+$是耦合波单位矢量。由(15)式、(18)式、(19)式、(16)式可得:

$$[\tilde{\Omega}_{p\pm}-\eta_1(\tilde{K}+\tilde{K}_{c,p1})-\tilde{K}_{p\pm}^2]\tilde{\boldsymbol{\psi}}_{c2}=-\frac{\tilde{N}\eta_2\tilde{\boldsymbol{\psi}}_c}{2} \quad (20)$$

$$[\pm\tilde{\Omega}_{c\pm}\mp\Delta\eta_1(\tilde{K}+\tilde{K}_{cp,1})-\tilde{K}_c^2\mp\eta_1\Delta\tilde{K}-2\tilde{K}_{p\pm}\tilde{K}_c]\tilde{\boldsymbol{\psi}}_{c2}=-\frac{\tilde{N}\Delta\eta_2\tilde{\boldsymbol{\psi}}_c}{2} \quad (21)$$

(21)式为修正方程;$\tilde{K}_{c,p1}=\tilde{K}_p\cos\theta$,$\theta$是$\tilde{\boldsymbol{K}}_p$与$\tilde{\boldsymbol{K}}_c$的夹角。由(17)式,可得:

$$-\frac{\tilde{N}}{2}=\tilde{\boldsymbol{\psi}}_c\cdot\tilde{\psi}_{c2}\tilde{\boldsymbol{e}}_{c2}^++\tilde{\boldsymbol{\psi}}_c^*\cdot\tilde{\psi}_{c2}\tilde{\boldsymbol{e}}_{c2} \quad (22)$$

由(20)式~(22)式,得:

$$\begin{aligned}&[\tilde{K}_p^4-(\tilde{\Omega}_p-\eta_1\tilde{K}_{c,p1}+2\tilde{K}_{c,p1}\tilde{K})]+\{[\tilde{K}_p^4-(\tilde{\Omega}_c-\Delta\eta_1\tilde{K}_{c,p1}-2\tilde{K}_{c,p1}\Delta\tilde{K})]+6\tilde{K}_p^2\tilde{K}_c^2+4\tilde{K}_p^3\tilde{K}_c+\\&4\tilde{K}_c^3\tilde{K}_p-2(\tilde{\Omega}_c\tilde{\Omega}_p-\eta_1\tilde{K}_{c,p1}\tilde{\Omega}_c-2\tilde{K}_{c,p1}\tilde{K}\tilde{\Omega}_c-\Delta\eta_1\Omega_p\tilde{K}_{c,p1}-\eta_1\Delta\eta_1\tilde{K}_{c,p1}^2+2\Delta\eta_1\tilde{K}_{c,p1}^2\tilde{K}+\\&2\tilde{\Omega}_p\tilde{K}_{c,p1}\Delta\tilde{K}+2\eta_1\tilde{K}_{c,p1}\Delta\tilde{K}+4\tilde{K}_{c,p1}^2\tilde{K}\Delta\tilde{K})\}\approx\eta_2|\boldsymbol{\psi}|^2\{[-\tilde{\Omega}_p+\eta_1\tilde{K}_{c,p1}-\tilde{K}_p^2+2\tilde{K}_{c,p1}\tilde{K})\sin^2\theta_++\\&(\tilde{\Omega}_p-\eta_1\tilde{K}_{c,p1}-\tilde{K}_p^2-2\tilde{K}_c\tilde{K}_p)\sin^2\theta_-]\}+\eta_2|\tilde{\boldsymbol{\psi}}_0|^2\{[\tilde{\Omega}_c+\Delta\eta_1\tilde{K}_{c,p1}-\tilde{K}_c^2+2\tilde{K}_{c,p1}\Delta\tilde{K}-\\&2\tilde{K}_c\tilde{K}_p)\sin^2\theta_++(\tilde{\Omega}_c-\Delta\eta_1\tilde{K}_{c,p1}\tilde{K}_c^2-2\tilde{K}_{c,p1}\tilde{K}-2\tilde{K}_{c,p1}\Delta\tilde{K})\sin^2\theta_-]\}+\Delta\eta_2|\psi|^2\times\\&\{[-\tilde{\Omega}_p+\eta_1\tilde{K}_{c,p1}-\tilde{K}_p^2-2\tilde{K}_{c,p1}\tilde{K})\sin^2\theta_++(\tilde{\Omega}_c-\eta_1\tilde{K}_{c,p1}-\tilde{K}_p^2-2\tilde{K}_c\tilde{K}_p)\sin^2\theta_-+\\&(\tilde{\Omega}_c-\tilde{K}_c^2-2\tilde{K}_c\tilde{K}_p)\sin^2\theta_++(\tilde{\Omega}_c-\tilde{K}_c^2-2\tilde{K}_{c,p1}\tilde{K})\sin^2\theta_-]\}-2\eta_2|\boldsymbol{\psi}\Delta\psi|\times\\&\{[\tilde{\Omega}_p-\eta_1\tilde{K}_{c,p1}+\tilde{K}_p^2-2\tilde{K}_{c,p1}\tilde{K})\sin^2\theta_+-(\tilde{\Omega}_p-\eta_1\tilde{K}_{c,p1}-\tilde{K}_p^2-2\tilde{K}_c\tilde{K}_p)\sin^2\theta_-]\}\end{aligned} \quad (23)$$

式中,$\theta_\pm$是$\tilde{K}_{c,p1}$与$\tilde{K}_c\pm\tilde{K}_p$夹角,等号左端第 2 项及右端第 2 项后的各项均为修正项。考虑自调制情况,若$\tilde{K}/\!/\tilde{K}_c$,则$\theta_\pm=\frac{\pi}{2}$。在有量纲制下,$(K_c^2+2K_cK_p+K_p^2)<\left(-\frac{4}{a_g}\right)(\beta|\boldsymbol{\psi}|^2+2\beta|\psi\Delta\psi|+\Delta\beta|\psi|)$,$(\beta+\Delta\beta)a_g<0$时,可得调制不稳定性时间增长率为:

$$\begin{aligned}\Gamma^2+\Gamma\Delta\Gamma\approx&-\frac{a_g^2}{4}K_p^4-\beta|\boldsymbol{\psi}|^2K_p^2-\frac{a_g^2}{2}\left(K_p^2K_c+\frac{3K_p^2K_c^2}{2}+K_pK_c^3\right)-\frac{a_g\Delta a_g}{4}(K_p^4+K_c^4+4K_p^3K_c+\\&6K_p^2K_c^2+4K_pK_c^3)-\beta|\boldsymbol{\psi}\Delta\boldsymbol{\psi}|(K_p^2+K_c^2)-\beta|\boldsymbol{\psi}|(K_pK_c+K_c^2)-\frac{\beta|\boldsymbol{\psi}|^2}{2}(K_p+K_c)^2\end{aligned} \quad (24)$$

式中,$\Gamma^2\approx-a_g^2K_p^4/4-\beta|\boldsymbol{\psi}|^2K_p^2$;$\Gamma\Delta\Gamma\approx-a_g^2\times(K_p^2K_c+3K_p^2K_c^2/2+K_pK_c^3)/2-a_g\Delta a_g(K_p^4K_c^4+4K_p^3K_c+6K_p^2K_c^2+4K_pK_c^3)/2-\beta|\boldsymbol{\psi}\Delta\boldsymbol{\psi}|(K_c^2+K_p^2)-|\boldsymbol{\psi}|(K_cK_p+K_c^2)-|\boldsymbol{\psi}|^2(K_c+K_p)^2/2$。$\Gamma\Delta\Gamma$表达式是时间增长率修正项。因式中各项均为负值,故散射使 SMI 最大时间增长率较散射前小。

4　数值模拟

考虑等离子体临界面附近局部表面区频率接近耦合光频率，SMI 时间增长率 $\frac{\Gamma_p+\Gamma_c}{\omega_c}$ 与扰动波数 $\frac{(\Gamma_p+\Gamma_c)c}{\omega_s-\omega}$ 关系如图 1 和图 2 所示。取入射光频率为 1.88×10^{15} Hz，入射光强度 0.9×10^{18} W/cm^2 ($|\tilde{\psi}|=0.850$)；散射光频率为 3.76×10^{15} Hz，散射光强度为 0.1×10^{18} W/cm^2 ($|\Delta\tilde{\psi}|=0.007$)；$\frac{\omega_p}{\omega_s-\omega}$ 为 0.5 和 0.9。图 1 和 2 中虚线和实线分别表示散射前后 MI 时间增长率与扰动波数关系曲线。由图 1 和 2 知，$\frac{\omega_p}{\omega_s-\omega}$ 越接近 1，即越靠近等离子体界面处，散射前后 MI 时间增长率增大越明显，且散射后比散射前增大更明显。可见，散射使 MI 发展比散射前来得快，激光峰值增加较强，并较快引起激光坍塌。

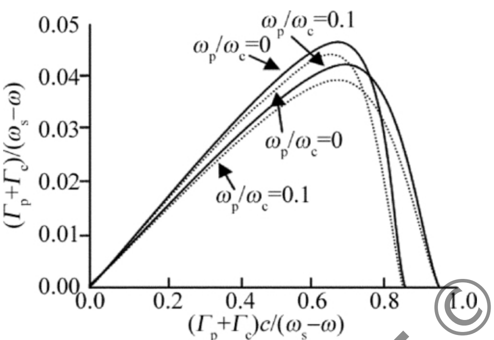

Fig. 1　Relation between the time rise rate of the self-modulation instability and perturbation wave numbers

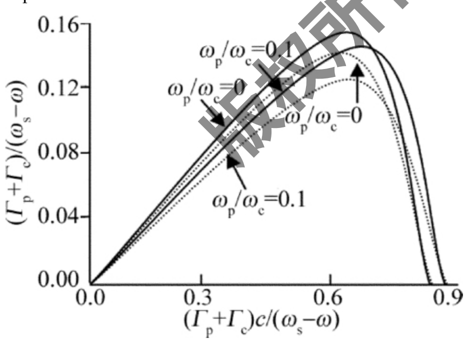

Fig. 2　Relation between the time rise rate of the self-modulation instability and perturbation wave numbers

5　结　论

(1) 散射分别使未磁化等离子体中和截面附近处 SMI 的最大时间增长率比散射前减小和增大。这是由于散射光使入射激光频率提高，等离子体界面附近的横等离子波、朗缪尔波和离子声波之间的碰撞频率及作用强度提高的缘故。

(2) 散射光的轴向磁场使左旋椭圆偏振光的衍射增强，导致激光自聚焦减弱。这主要是由于散射引起的扰动不可忽略，使调制不稳定性出现非线性发展，导致激光场坍塌的缘故。

可见，实验中可通过控制入射激光强度实现对未磁化激光等离子体中调制不稳定性的调控。

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