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闪耀光栅的衍射特性研究

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摘要： 为了系统研究闪耀光栅的衍射特性，分析光栅结构参量对其衍射效率的影响，采用严格的耦合波分析方法，得到了闪耀光栅的槽顶角、闪耀角等结构参量以及波长与衍射效率的关系。结果表明，随着槽顶角的增大，闪耀角向减少的方向移动，衍射效率则向降低的方向移动；槽顶角一定时，随着光波长的增大，衍射效率降低，同时闪耀角向增大的方向移动；同时分析了闪耀光栅的偏振现象。分析结果可为闪耀光栅的设计提供参考依据。

关键词： 衍射与光栅；衍射效率；严格耦合波分析；光栅结构参量
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Analysis of diffraction characteristics of blazed gratings

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Abstract: In order to study the diffraction characteristics of blazed gratings and analyze the influence of grating parameters on diffraction efficiency, the dependence of diffraction efficiency on the different grating structure parameters and wavelengths was obtained based on the rigorous coupled-wave electromagnetic theory. Calculation results showed that the larger the slot angle, the smaller the blazed angle and the diffraction efficiency, and the bigger the wavelength, the smaller the diffraction efficiency and the bigger the blazed angle. Finally, the polarization phenomenon of a blazed grating was analyzed. The obtained results are helpful for the grating design.

Key words: diffraction and grating; diffraction efficiency; rigorous coupled-wave analysis; grating parameter

引　言

衍射光栅广泛应用于光谱学、激光核聚变、无线电天文学、集成光学、光通讯、信息处理及精密计量等不同领域[1-5]，衍射光栅最重要的指标之一是光栅的衍射效率。为提高光栅的衍射效率，有学者研究了光栅槽形对光强分布的影响，提出了“闪耀光栅”理论[6]。闪耀光栅是一种能将单个刻槽面衍射的中央极大和诸槽面间干涉零级主极大分开的光栅，它的刻槽面与光栅面不平行，两者之间有一夹角（称为闪耀角），从而使单个刻槽面（相当于单缝）衍射的中央极大和诸槽面间（缝间）干涉零级主极大分开，将光能量从干涉零级主极大，即零级光谱，转移并集中到某一级光谱上去，实现该级光谱的闪耀[7]。在闪耀光栅的应用中，准确地分析计算光栅的衍射效率特性具有非常重要的实际意义，分析光栅衍射特性的严格理论可分为两大类：积分方法[7]和微分方法[8]。在微分方法中，严格的耦合波理论具有物理概念清晰、易于稳定的数值模拟计算等特点[9-13]。作者以严格的耦合波理论为研究手段，以朗姆斯特（Lamost）望远镜用透射闪耀光栅为研究对象，研究和分析了闪耀光栅的槽顶角、闪耀角以及波长的变化对其衍射效率的影响规律。

1　闪耀光栅的严格耦合波分析

闪耀光栅的衍射问题如图1所示。闪耀光栅的槽顶角为 α，闪耀角为 β，光栅周期为 Λ，槽深为 d。以TE模式为例，电磁波以入射角 θ 入射，则光栅将产生各级反射波及透射波。反射区域及透射区域的介电常分别为 ε_1 和 ε_2，光栅区域包含了两种介质的周期分布，其介电常数为一周期函数，采用严格的耦合波方法分析时将光栅分为 N 层薄层，设第 n 层光栅的深度为 d_n。则每一个薄层光栅的介电常数为一周期函数，可用傅里叶级数展开为：

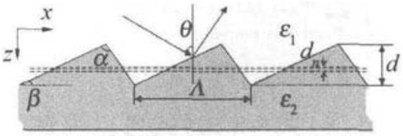

Fig. 1　The scheme of a blazed grating

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$$\varepsilon_n(x) = \sum_{i=-\infty}^{\infty}\varepsilon_{n,i}\exp(\mathrm{j}\frac{2\pi i}{\Lambda}x) \tag{1}$$

式中，$\varepsilon_{n,i}$为第 n 薄层光栅介电常数的傅里叶级数的第 i 级系数，它是一个常数。

对于TE偏振入射光，电场矢量平行于刻槽方向，入射平面波的波长为 λ，在入射区及透射区，介电常数为常数，可以直接求得在这两个区域内的电场分布为：

$$E_1 = \exp[-\mathrm{j}(k_{0x}x - k_{0z}z)] + \sum_{i=-\infty}^{\infty}R_i\exp[-\mathrm{j}(k_ix - k_{1zi}z)] \tag{2}$$

$$E_3 = \sum_{i=-\infty}^{\infty}T_i\exp\{-\mathrm{j}[k_ix - k_{3zi}(z-d)]\} \tag{3}$$

式中，$k_{0x} = k_0\sqrt{\varepsilon_1}\sin\theta$；$k_{0z} = k_0\sqrt{\varepsilon_1}\cos\theta$；$k_0 = 2\pi/\lambda$；$\lambda$ 为自由空间光波长。$k_i = k_{0x} - ik_0\lambda/\Lambda$；$k_i$ 为第 i 级衍射波矢的 x 分量。k_{1zi}，k_{3zi}分别为入射区、透射区域第 i 级衍射波矢的 z 分量；R_i，T_i 分别为第 i 级反射衍射波、透射波的振幅；在光栅区域，第 n 薄层电场、磁场可以表示为空间谐波的傅里叶级数展开：

$$\begin{cases} E_{n,2y} = \sum_{i=-\infty}^{\infty}V_{n,i}(z)\exp(-\mathrm{j}k_ix) \\ H_{n,2x} = -\mathrm{j}\sqrt{\varepsilon_0/\mu_0}\sum_{i=-\infty}^{\infty}U_{n,i}(z)\exp(-\mathrm{j}k_ix) \end{cases} \tag{4}$$

$U_{n,i}(z)$，$V_{n,i}(z)$分别为第 n 层第 i 级电场、磁场空间谐波的振幅；在第 n 层光栅区域，电磁场必须满足麦克斯韦基本方程组：

$$\begin{cases} \frac{\partial E_{n,2y}}{\partial z} = \mathrm{j}\omega\mu_0H_{n,2x} \\ \frac{\partial H_{n,2x}}{\partial z} = \mathrm{j}\omega\varepsilon_0\varepsilon(x)E_{n,2y} + \frac{\partial H_{n,2z}}{\partial x} \end{cases} \tag{5}$$

将(4)式分别代入(5)式，经过一系列数学推导，得到第 n 层光栅区域的一组耦合波方程：

$$\frac{\partial^2V_{n,i}}{\partial z^2} = k_iV_{n,i} - k_0^2\sum_{m=-\infty}^{\infty}\varepsilon_{n,i-m}V_{m,n} \tag{6}$$

耦合波方程(6)式可采用本征值法求解：

$$\begin{cases} V_{n,i}(z) = \sum_m w_{n,lm}\{c_{n,m1}\exp[-k_0q_{n,m}(z - D_n + d_n)] + c_{n,m2}\exp[k_0q_{n,m}(z - D_n)]\} \\ U_{n,i}(z) = \sum_m s_{n,lm}\{-c_{n,m1}\exp[-k_0q_{n,m}(z - D_n + d_n)] + c_{n,m2}\exp[k_0q_{n,m}(z - D_n)]\} \end{cases} \tag{7}$$

式中，$D_n - d_n < z < D_n$，$D_n = \sum_{l=1}^{n}d_l$，$q_{n,m}$为本征值；$w_{n,lm}$是本征值$q_{n,m}$对应的本征矢量的第l个分量；$s_{n,lm} = w_{n,lm}q_{n,m}$；$c_{n,m1}$，$c_{n,m2}$为待定系数，由边界条件来确定。

在入射区域与第1薄层光栅的边界面上，有：

$$\begin{cases} \delta_{i0} + R_i = \sum_m w_{1,lm}[c_{1,m1} + c_{1,m2}\exp(-k_0q_{1,m}d_1)] \\ \mathrm{j}[k_{0z} - k_{1zi}R_i] = \sum_m s_{1,lm}k_0[c_{1,m1} - c_{1,m1}\exp(-k_0q_{1,m}d_1)] \end{cases} \tag{8}$$

在第 N 薄层光栅与透射区域的边界面上，有：

$$\begin{cases} T_i = \sum_m w_{N,lm}[c_{N,m2} + c_{N,m1}\exp(-k_0q_{1,m}d_N)] \\ \mathrm{j}k_{3zi}T_i = -k_0\sum_m s_{N,lm}[c_{N,m2} + c_{N,m1}\exp(-k_0q_{1,m}d_N)] \end{cases} \tag{9}$$

在第 n 薄层光栅与第 $n-1$ 薄层光栅的边界面上有：

$$\begin{cases} \sum_m w_{n-1,lm}[c_{n-1,m1}\exp(-k_0q_{n-1,m}d_n) + c_{n-1,m2}] = \sum_m w_{n,lm}[c_{n,m1} + c_{n,m2}\exp(-k_0q_{n,m}d_n)] \\ \sum_m s_{n-1,lm}[c_{n-1,m1}\exp(-k_0q_{n-1,m}d_n) - c_{n-1,m2}] = \sum_m s_{n,lm}[c_{n,m1} - c_{n,m2}\exp(-k_0q_{n,m}d_n)] \end{cases} \tag{10}$$

直接由方程组(8)式~(10)式可以求解 $c_{n,m1}$，$c_{n,m2}$及R_i，T_i，为提高计算速度及保证计算过程的数值稳定性，可以采用增强透射矩阵方法进行求解[10]。则反射区及透射区的各级衍射波的衍射效率为：

$$\begin{cases} \eta_{r,i} = |R_i|^2\mathrm{Re}\left[\frac{k_{1zi}}{k_{0x}}\right] \\ \eta_{t,i} = |T_i|^2\mathrm{Re}\left[\frac{k_{3zi}}{\varepsilon_2}\right]\Big/\left(\frac{k_{0x}}{\varepsilon_1}\right) \end{cases} \tag{11}$$

2 Lamost望远镜用透射光栅的衍射特性

利用耦合波方法对Lamost望远镜用透射光栅的衍射特性作了深入细致的计算与分析。给出了闪耀角、槽顶角等光栅特征尺寸对光栅效率特性的影响，本文中讨论的透射闪耀光栅如图2所示。光栅槽密度为600/mm，工作波段分别为0.39μm~0.61μm与0.59μm~0.9μm。

首先，图3a和图3b中分别模拟了TE波和TM波入射情况下，光栅在相同的光栅闪耀角$\beta = 22.35°$，而槽顶角分别为$\alpha = 90°$及$\alpha = 110°$时效率与波长的关系曲线。由图可知，随着波长的增加偏振显现，但并不明显。

图4和图5中给出了在不同波段光栅的工作特性。图4中首先给出了0.39μm~0.61μm、0.59μm~

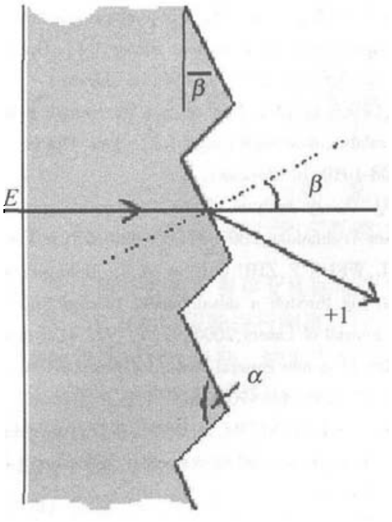

Fig. 2　Geometry of a grating

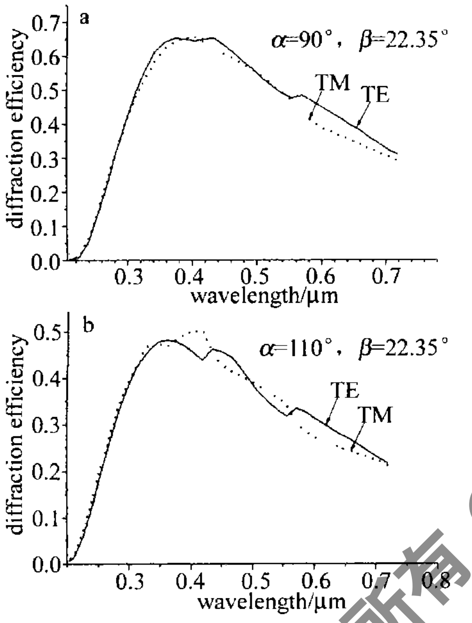

Fig. 3　The relation between the diffraction efficiency and the wavelength for TE and TM

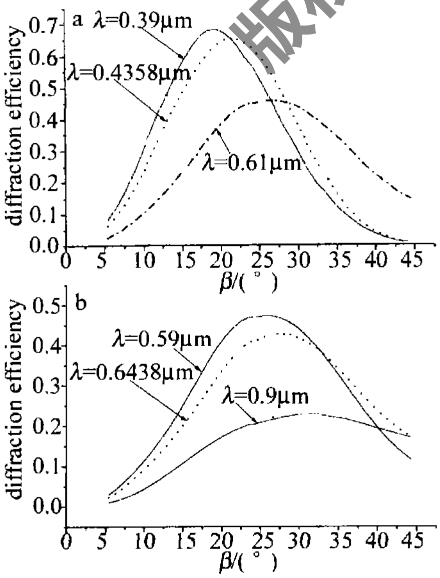

Fig. 4　The relation between the diffraction efficiency and blazed angle, $\alpha = 90^\circ$

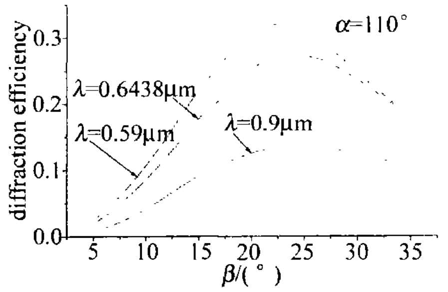

Fig. 5　The relation between the diffraction efficiency and blazed angle for different wavelengths

0.9μm波段内，在槽顶角为 $\alpha = 90^\circ$时不同波长的情况下，衍射效率与闪耀角的关系特性曲线。可见，光栅的闪耀角分别在18°～27°、25°～32°变化，且波长愈短，闪耀角愈小，衍射效率愈高；$\alpha = 110^\circ$时，衍射效率与闪耀角的变化关系遵循相似的规律，以0.59μm～0.9μm波段的工作特性为例，图5中给出了波长不同时，衍射效率与闪耀角的关系特性曲线，闪耀角在22°～26°变化。

图6中给出了在槽顶角不同而波长相同的情况下，衍射效率与闪耀角的关系特性曲线，由图可见，随着槽顶角的增大，闪耀角向减少的方向移动，衍射效率则向降低的方向移动；槽顶角的大小对光栅衍射效率的影响较大，可以根据光栅刻划面积的大小选择适当

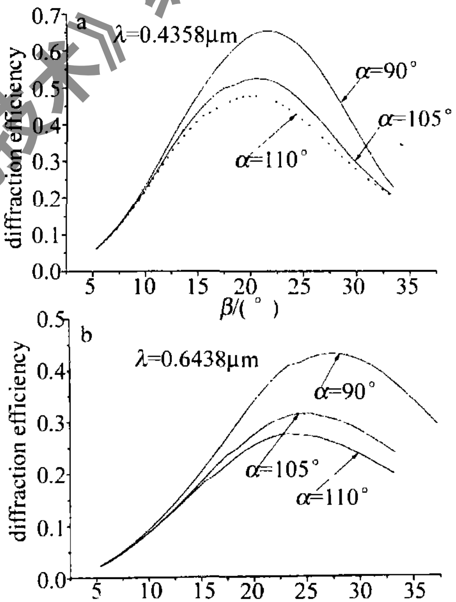

Fig. 6　The relation between the diffraction efficiency and blazed angles

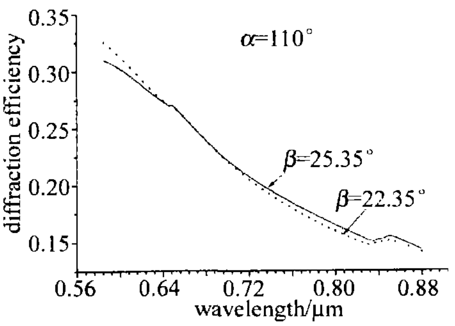

Fig. 7　The relation between the diffraction efficiency and wavelength for different blazed angles

的金刚石刀具。

图7中给出了槽顶角 $\alpha = 110°$ 时闪耀角不同的情况下，衍射效率与波长的关系特性曲线。

由图8可知，衍射效率随着波长的增加成下降趋势，并且槽顶角越大，衍射效率越低。

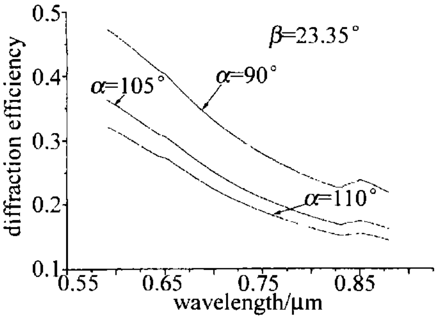

Fig. 8 The relation between the diffraction efficiency and wavelength for different angles

3 结论

采用严格的耦合波分析方法对透射式闪耀光栅的衍射效率特性进行了研究与分析。用增强透射矩阵方法求解的耦合波方法求解闪耀光栅的衍射特性，计算稳定、求解效率高。给出了光栅结构参量、入射波长等对光栅衍射效率的影响，研究结果对闪耀光栅的设计具有指导意义和参考价值。

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