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贝塞尔-高斯光束通过圆孔与圆环光阑的衍射

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摘要: 为了研究贝塞尔-高斯光束通过圆孔硬边光阑和圆环光阑的衍射特性, 从 Collins公式出发, 采用数值模拟的方法模拟出光强分布。模拟结果表明, 贝塞尔-高斯光束经圆孔光阑衍射后轴上光强随菲涅耳数 F 呈周期振荡; 贝塞尔-高斯光束经圆环光阑后轴上光强随 F 呈振动衰减。在 F 相同时, 贝塞尔-高斯光束经圆孔光阑衍射后横向光强分布比经圆环光阑衍射后横向光强分布平滑, 孔径越小, 光强调制越明显; 当孔径与束腰相等时候, 横向光强分布与菲涅耳数没有关系。

关键词: 激光光学; 贝塞尔高斯光束; 衍射; 圆孔光阑; 圆环光阑

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Diffraction of Bessel-Gaussian beam passing through annular and circular apertures

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Abstract: From the Collins formula, the diffraction of Bessel-Gaussian beam passing through circular and annular apertures was studied based on numerical simulation. The results indicate that the axial intensity of Bessel-Gaussian beam through a circular aperture changes with the Fresnel number F periodically; the axial intensity of Bessel-Gaussian beam through an annular aperture attenuates with the F at the function of vibration. When F is the same, the transversal intensity of Bessel-Gaussian beam through a circular aperture distributes more smooth than an annular aperture. The smaller the annular aperture, more obvious is the modulation of transversal intensity; when the annular aperture is equal to the waist of the beam, the transversal intensity is independent of the Fresnel number F.

Key words: laser optics; Bessel-Gaussian beam; diffraction; annular aperture; circular aperture

引 言

贝塞尔-高斯光束是一种有应用前景的光束, 它在一定条件下呈现“无衍射”特性, 对这种光束的研究引起人们的极大关注[1-10]。LIU 等人对贝塞尔光束及贝塞尔-高斯光束的传输和聚焦特性已做了详细的计算分析和实验研究进行了比较[5]; GREENE, OVERFELT 等人对贝塞尔-高斯光束经不同几何构形光阑的衍射作了比较研究[6-7]; JIANG等人计算了加光阑贝塞尔光束的空间频谱[8]。作者就贝塞尔-高斯光束经圆孔光阑和圆环光阑衍射后光强分布随菲涅耳数 F 的变化作了研究, 并对 F 相同时的横向光强分布, 以及当孔径与束腰相等时的横向光强分布与菲涅耳数的关系作了比较, 对进一步研究贝塞尔-高斯光束有理论和现实意义。

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1 贝塞尔-高斯光束通过硬边光阑的衍射理论

1.1 理论分析

贝塞尔-高斯光束在 $z=0$处的场分布为[5]:

$$E_0(r_0, 0) = \mathrm{J}_0(\alpha r_0)\exp(-r_0^2/w_0^2) \tag{1}$$

式中, α为横向波数, w_0为束腰半径, J_0为第一类零阶贝塞尔函数。贝塞尔-高斯光束经过透过率为 $T(r_0, \theta_0)$的光阑衍射后的场分布可由 Collins公式描述为:

$$E(r, \theta, z) = \left(-\frac{\mathrm{i}}{\lambda B}\right)\iint E(r_0, \theta_0, 0)T(r_0, \theta_0)\times \exp\left\{\frac{\mathrm{i}k}{2B}[Ar_0^2 - 2r_0 r\cos(\theta-\theta_0) + Dr^2]\right\}r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0 \tag{2}$$

式中, z为观察面与光阑的距离, (r_0, θ_0)和(r, θ)分别为光阑面和观察面的极坐标, k为波矢。

自由空间的传输矩阵 $ABCD$ 满足:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \tag{3}$$

把(3)式代入(2)式, 得:

$$E(r, \theta, z) = \left(-\frac{\mathrm{i}}{\lambda z}\right)\iint T(r_0, \theta_0)\mathrm{J}_0(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right)\times$$

$$\exp\left\{\frac{ik}{2z}\left[r_0^2 + r^2 - 2rr_0\cos(\theta_0 - \theta)\right]\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0 \quad (4)$$

1.2 圆孔硬边光阑

圆孔透过率为:

$$T(r_0,\theta_0) = \left(\frac{r_0}{w_0}\right)_{\text{circular}} = \begin{cases} 1, & (r_0 \leqslant a_0) \\ 0, & (r_0 > a_0) \end{cases} \quad (5)$$

式中, circular表示圆孔, a_0 为圆孔光阑半径, 将(5)式代入(4)式, 可得贝塞尔-高斯光束经过圆孔光阑衍射后的场分布:

$$E(r,\theta,z) = \left(-\frac{i}{\lambda z}\right)\int_0^{a_0}\int_0^{2\pi} J_0(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right) \times$$

$$\exp\left\{\frac{ik}{2z}\left[r_0^2 + r^2 - 2rr_0\cos(\theta_0 - \theta)\right]\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0 \quad (6)$$

上式对 θ_0 积分, 得:

$$E(r,\theta,z) = \left(-\frac{2\pi}{\lambda z}\right)i\int_0^{a_0} J_0(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right) \times$$

$$\exp\left[\frac{ik}{2z}(r_0^2 + r^2)\right] J_0\left(\frac{ik}{2z}rr_0\right) r_0\,\mathrm{d}r_0 \quad (7)$$

上述计算中用到公式:

$$J_n(x) = \frac{1}{2\pi}\int_0^{2\pi}\exp\left[ix\cos(\theta - \varphi) + in\left(\theta - \frac{\pi}{2}\right)\right]\mathrm{d}\theta \quad (8)$$

式中, $J_n(x)$为第 1类 n阶贝塞尔函数, x 为变量, n 为阶数。

令 $\rho = \frac{r}{a_0}$, $\rho_0 = \frac{r_0}{a_0}$, 菲涅耳数 $F = \frac{a_0^2}{\lambda z}$, 可得:

$$E(\rho,F) = (-i2\pi F)\exp(-i\pi F\rho^2)\int_0^1 J_0(\alpha a_0\rho_0) \times$$

$$J_0(2\pi F\rho\rho_0)\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0 \quad (9)$$

由 $E(\rho,F)$可得贝塞尔-高斯光束经过圆孔衍射后在轴上场分布为:

$$E(0,F) = (-i2\pi F)\int_0^1 J_0(\alpha a_0\rho_0) \times$$

$$\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0 \quad (10)$$

$$I(\rho,F) = |E(\rho,F)|^2 = (2\pi F)^2\left|\int_0^1 J_0(\alpha a_0\rho_0) \times\right.$$

$$\left. J_0(2\pi F\rho\rho_0)\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0\right|^2 \quad (11)$$

$$I(0,\rho) = |E(0,F)|^2 = (2\pi F)^2\left|\int_0^1 J_0(\alpha a_0\rho_0) \times\right.$$

$$\left.\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0\right|^2 \quad (12)$$

1.3 圆环硬边光阑

圆环透过率为:

$$T(r_0,\theta_0) = \left(\frac{r_0}{a_0}\right)_{\text{annular}} = \begin{cases} 1, & (a_0 \leqslant r_0 \leqslant 2a_0) \\ 0, & (r_0 > 2a_0,\ r_0 < a_0) \end{cases} \quad (13)$$

式中, a_0, $2a_0$分别为圆环的内外半径。将(13)式代入(4)式, 可得贝塞尔-高斯光束经过圆环光阑衍射后的场分布:

$$E(\rho,F) = (-i2\pi F)\exp(-i\pi F\rho^2)\int_1^2 J_0(\alpha a_0\rho_0) \times$$

$$J_0(2\pi F\rho\rho_0)\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0 \quad (14)$$

$$E(r,\theta,z) = \left(-\frac{i}{\lambda z}\right)\exp(ikz)\int_{a_0}^{2a_0}\int_0^{2\pi} J_0(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right) \times$$

$$\exp\left\{\frac{ik}{2z}\left[r_0^2 + r^2 - 2rr_0\cos(\theta_0 - \theta)\right]\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0 \quad (15)$$

上式对 θ_0 积分, 得:

$$E(r,\theta,z) = \left(-\frac{2\pi}{\lambda z}\right)i\int_{a_0}^{2a_0} J_0(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right) \times$$

$$\exp\left[\frac{ik}{2z}(r_0^2 + r^2)\right] J_0\left(\frac{ik}{z}rr_0\right) r_0\,\mathrm{d}r_0 \quad (16)$$

$$E(\rho,F) = (-i2\pi F)\exp(-i\pi F\rho^2)\int_1^2 J_0(\alpha a_0\rho_0) \times$$

$$J_0(2\pi F\rho\rho_0)\exp\left[\left(i\pi F - \frac{a_0^2}{w_0^2}\right)\rho_0^2\right]\rho_0\,\mathrm{d}\rho_0 \quad (17)$$

2 模拟和分析

用 MAPLE 软件进行模拟, 得到贝塞尔-高斯光束($\alpha = 10\mathrm{mm}^{-1}$, $w_0 = 1\mathrm{mm}$)经两种不同几何构形光阑衍射后具体的光强分布图形, 分别见图 1、图 2。由图 1可见, 贝塞尔-高斯光束经圆孔光阑衍射后轴上光强随菲涅耳数 F 呈周期振动, 经圆环衍射后轴上光强随菲涅耳数 F 呈振动衰减。

同样, 还研究了在菲涅耳数 $F = 20$的时候, 贝塞

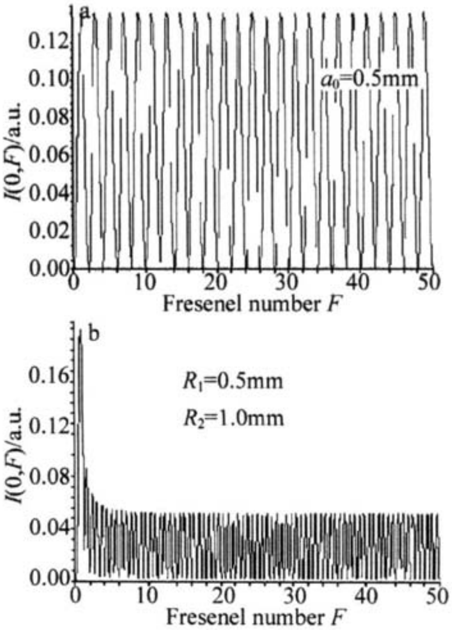

Fig 1 Axial intensity distributions of a Bessel-Gaussian beam diffracted by different apertures versus Fresnel number F

a—circular aperture b—annular aperture

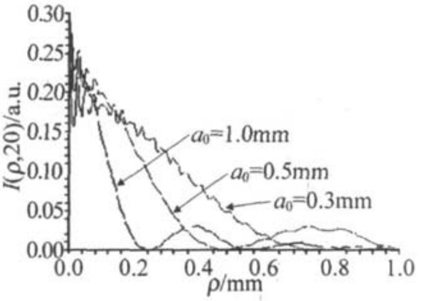

Fig. 2　The relation between transverse intensity distributions of a Bessel-Gaussian beam diffracted by circular apeture as $F = 20$

尔-高斯光束经不同孔径圆孔光阑衍射后横向光强随孔径的变化关系，见图 2。可见，孔径相对小时，横向光强的分布调制得越厉害；但是当孔径为 1mm 时，衍射光强随着菲涅耳数 F 的变化而没有多大变化，这说明了贝塞尔-高斯光束的无衍射特性，见图 3。

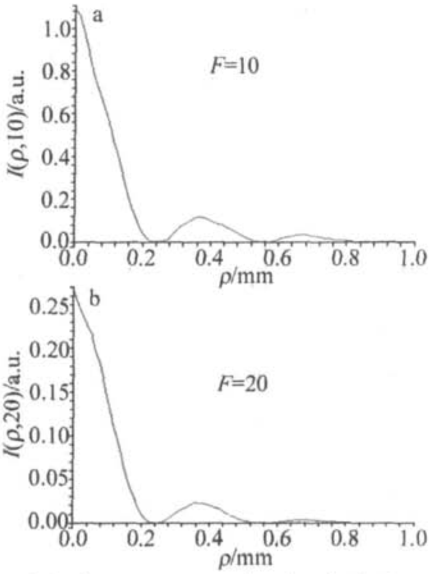

Fig. 3　The relation between transverse intensity distributions of a Bessel-Gaussian beam and Fresnel number F as circular aperture equal to 1mm

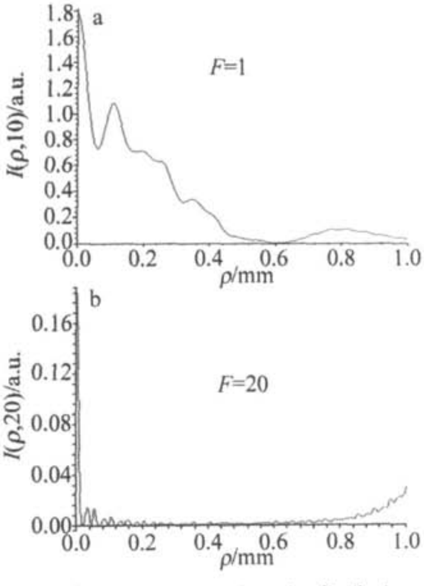

Fig. 4　The relation between transverse intensity distributions of a Bessel-Gaussian beam by annular aperture and Fresnel number F

由图 4可见，贝塞尔-高斯光束经圆环光阑衍射后横向光强随菲涅耳数 F 增大不均匀度随之增大，光强随 ρ的增大而减小，但减小到一定程度时略有上升，也可见该种光束的近场无衍射特性。

3　小　结

从 Collins公式出发，对贝塞尔-高斯光束经硬边光阑的衍射作了研究，给出了贝塞尔-高斯光束经硬边光阑后的轴上光强分布和横向光强分布。通过数值计算模拟出光强分布，对贝塞尔-高斯光斯光束经圆孔光阑和圆环光阑后光强作了比较。在菲涅耳数 F 相同时，贝塞尔-高斯光束经圆孔光阑衍射后横向光强分布比经圆环光阑衍射后横向光强分布平滑，孔径越小，光强调制越厉害；但是当孔径与束腰相等时候，横向光强分布与菲涅耳数没有关系，光束为直线传播形式，亦即该种光束的发散角几乎为 0，也就很容易地解释了贝塞尔-高斯光束的无衍射特性，对进一步研究贝塞尔-高斯光束有理论和现实意义。

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