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# 高阶色散下随入纤功率变化的不稳定性增益谱

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**摘要:** 为了探讨不同入纤功率下高阶色散对交叉相位调制不稳定性增益谱的影响, 从光纤中包含高阶色散的耦合非线性薛定谔方程出发, 采用线性稳定性分析法, 分二阶、四阶色散系数同号、异号和二阶色散系数为 0 等几种情形, 计算了双光束的交叉相位调制不稳定性增益谱随入纤光功率的变化规律, 并对各种谱特性的产生机制作了分析。结果表明, 当二阶、四阶色散系数同号时, 随着入纤功率的增大, 增益谱由开始的两个分离谱区逐渐变宽并合成 1 个谱区; 当二阶、四阶色散系数异号和二阶色散系数为 0 时, 增益谱只有靠近零点的第一谱区, 且谱宽和谱峰随着入纤功率的增大而增大。此研究对高重复率的超短光脉冲串的产生有一定的理论指导意义。

**关键词:** 非线性光学; 交叉相位调制不稳定性; 耦合非线性薛定谔方程; 高阶色散; 增益谱

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## Modulation instability gain spectrum varying with the incident optical power in case of high-order dispersion

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**Abstract** In order to investigate the effect of the high order dispersion on the gain spectra of cross phase modulation (XPM) instability under different incident optical power, starting from the coupled nonlinear Schrödinger equations of two optical waves in an optical fiber and utilizing the linear order stability analysis, the gain spectra of cross phase modulation instability varying with the incident optical power was calculated when the second-order and the fourth-order dispersion coefficients have the same opposite signs and when the second-order dispersion coefficient was equal to zero, respectively. The mechanism behind these diverse spectra was analyzed in detail. The results show that when the second-order and the fourth-order dispersion coefficients have the same sign with the increase of the incident optical power, the gain spectrum which consists of two separated regions first will broaden and combine to one region. When the second-order and the fourth-order dispersion coefficients have the opposite signs and the second-order dispersions are equal to zero, the gain spectrum consists of only the first spectral region near the zero point. Moreover, the width and the peak value of the gain spectrum will increase with the incident optical power. The investigation can be a theory guidance to generate ultrashort optical pulse chains with high repetition rate to some extent.

**Key words** nonlinear optics; cross phase modulation instability; coupled nonlinear Schrödinger equations; high order dispersion; gain spectra

## 引言

由于光纤中的调制不稳定性在高重复率的超短光脉冲串<sup>[1]</sup>、超宽带连续谱的产生<sup>[2]</sup>等方面的应用以及对波分复用(wavelength-division multiplexing WDM)或密集波分复用(dense wavelength-division multiplexing

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DWDM)光纤通信系统的严重损害<sup>[3~4]</sup>, 人们已从多方面对该现象进行研究。

单光束的自相位调制不稳定性在低阶和高阶色散下均得到了广泛的研究<sup>[5~8]</sup>, 但对双光束的交叉相位调制不稳定性研究则还基本上限于低阶色散的情况<sup>[9~11]</sup>, 高阶色散下的研究极少<sup>[12]</sup>。而缩短光脉冲的脉宽对于提高光纤通信系统的速度和容量是有益的。但是, 当光脉冲的脉宽较窄, 或其载频波长位于光纤零色散波长附近时, 高阶色散的影响不可忽略。REN 等人<sup>[12]</sup>首次对高阶色散下双光束的交叉相位调制不稳定性谱特性和机制作了研究, 但研究仅限于两光束的二阶、四阶色散系数同号的情况, 不够全面, 也未深入分析入纤功率变化对各色散区的增益谱的影

响及影响机制。

作者在文献 [12] 的基础上, 从更完整和深入的角度出发, 进一步就二阶和四阶色散系数同号、异号和二阶色散系数为 0 几种情形, 计算了交叉相位调制不稳定性增益谱随入纤光功率的变化规律, 并对出现各种谱特性的机制作了详细分析。

## 1 理论分析

当同偏振、不同波长的两光波在包含二阶至四阶色散和损耗的光纤中传输时, 需满足下列扩展的耦合非线性薛定谔方程<sup>[3-12]</sup>:

$$\frac{\partial A_j}{\partial z} + \frac{1}{v_{gj}} \frac{\partial A_j}{\partial t} + \frac{i}{2} \beta_{2j} \frac{\partial^2 A_j}{\partial t^2} - \frac{\beta_{3j}}{6} \frac{\partial^3 A_j}{\partial t^3} + \frac{i}{24} \beta_4 \frac{\partial^4 A_j}{\partial t^4} + \frac{\alpha_j}{2} A_j = \gamma_j (|A_j|^2 + 2|A_{3-j}|^2) A_j \quad (1)$$

式中,  $A_j$  ( $j = 1, 2$ ),  $v_{gj}$ ,  $\beta_{mj}$  ( $m = 2, 3, 4$ ) 和  $\gamma_j$  分别表示两光波的慢变振幅、群速度、 $m$  阶群速度色散系数和三阶非线性系数,  $\alpha_j$  为光纤的损耗系数,  $t$  和  $z$  分别为时间和传输距离。当两光波的波长差异很小时, 可认为  $v_{g1} \approx v_{g2} = v_g$ <sup>[3-12]</sup>,  $\beta_{1i} \approx \beta_{i2} = \beta_i$ <sup>[3-12]</sup> ( $i = 2, 3, 4$ ), 采用人们熟知的线性稳定性分析法<sup>[3-7]</sup>可得到扰动波数  $k$  满足的色散关系为:

$$k = \frac{\Omega}{v_g} + \frac{1}{6} \beta_3 \Omega^3 \pm \left\{ \frac{f_1 + f_2}{2} \pm \left[ \left( \frac{f_1 + f_2}{2} \right)^2 + (C_{XPM} - f_1 f_2) \right]^{1/2} \right\}^{1/2} \quad (2)$$

式中,  $f_i = \left( \frac{1}{2} \beta_{2i} \Omega^2 - \frac{1}{24} \beta_{4i} \Omega^4 \right) \times \left[ \frac{1}{2} \beta_{2i} \Omega^2 - \frac{1}{24} \beta_{4i} \Omega^4 + 2\gamma_i P_i \exp(-\alpha_i z) \right]$  (3)

$\Omega$  为光扰动的角频率,  $P_i$  为两光波的入纤功率, 交叉相位调制耦合参量  $C_{XPM}$  满足:

$$C_{XPM} = 16\gamma_1 \gamma_2 P_1 P_2 \exp(-\alpha_1 z) \exp(-\alpha_{3-j} z) \times \left( \frac{1}{2} \beta_{21} \Omega^2 - \frac{1}{24} \beta_{41} \Omega^4 \right) \left( \frac{1}{2} \beta_{22} \Omega^2 - \frac{1}{24} \beta_{42} \Omega^4 \right) \quad (4)$$

对于频率满足不等式  $f_1 f_2 < C_{XPM}$  的那些扰动,  $k$  成为复数, 调制不稳定性产生。扰动的功率增益系数为:

$$g(\Omega) = 2 \operatorname{Im}(k) = \sqrt{2} \{ [(f_1 + f_2)^2 + 4(C_{XPM} - f_1 f_2)]^{1/2} - (f_1 + f_2) \}^{1/2} \quad (5)$$

由 (3) 式 ~ (5) 式可见, 交叉相位调制不稳定性条件和增益谱只受二阶和四阶色散的影响, 而不受三阶色散的影响, 但由 (2) 式可见, 三阶色散要影响扰动波数  $k$  的实部。为简单起见, 下面讨论中将忽略损耗的影响。

## 2 计算与讨论

由于三阶色散不影响交叉相位调制不稳定性的情

件和增益谱, 所以下面的讨论中不再涉及。

图 1a 和图 1b 中分别给出了二阶、四阶色散系数同为正和负时, 交叉相位调制不稳定性增益谱随入纤

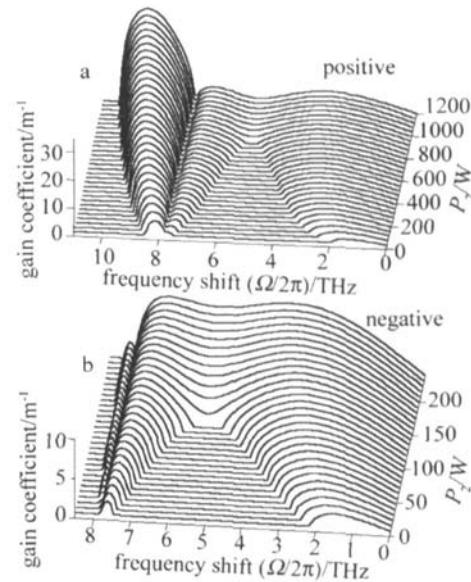


Fig. 1 Variation of gain spectra with the incident optical power  $P_2$  when the second-order and the fourth-order dispersion coefficients are both positive or negative

功率  $P_2$  的变化情况。图 1a 中的参数为:  $\beta_2 = 20 \text{ ps}^2 / \text{km}$ ,  $\beta_4 = 0.1 \text{ ps}^4 / \text{km}$ ,  $\gamma = 10 \text{ W/km}$ ,  $P_1 = 150 \text{ W}$ 。图 1b 中的参数为:  $\beta_2 = -20 \text{ ps}^2 / \text{km}$ ,  $\beta_4 = -0.1 \text{ ps}^4 / \text{km}$ ,  $\gamma = 10 \text{ W/km}$ ,  $P_1 = 60 \text{ W}$ 。由于  $g(-\Omega) = g(\Omega)$ , 各图中均只画出了  $\Omega > 0$  的部分。

由图可见, 当  $P_2$  较小时, 增益谱由两个分离的谱区构成, 与文献 [12] 中的结果一致。可将靠近零点的谱区称为第 1 谱区, 远离零点的称为第 2 谱区, 第 2 谱区由两个相连的小谱区构成, 在  $\beta_2 > 0$  的正色散区, 靠近零点的小谱区较窄, 谱峰较小, 而在负色散区, 情况相反。随着  $P_2$  的增大, 两谱区均变宽且二者的间距缩短, 当  $P_2$  增大到一定时候, 二谱区合二为一。相比较而言, 负色散区达到二谱区合二为一所需的功率  $P_2$  要小些。

图 2a 和图 2b 中分别给出了二阶色散为正、四阶色散为负和二阶色散为负、四阶色散为正时的增益谱随入纤功率  $P_2$  的变化规律。图 2a 中,  $\beta_2 = 20 \text{ ps}^2 / \text{km}$ ,  $\beta_4 = -0.1 \text{ ps}^4 / \text{km}$ ,  $\gamma = 10 \text{ W/km}$ ,  $P_1 = 150 \text{ W}$ 。图 2b 中,  $\beta_2 = -20 \text{ ps}^2 / \text{km}$ ,  $\beta_4 = 0.1 \text{ ps}^4 / \text{km}$ ,  $\gamma = 10 \text{ W/km}$ ,  $P_1 = 60 \text{ W}$ 。可见, 此时的增益谱只有靠近零点的第 1 谱区, 随着  $P_2$  的增大, 谱宽和谱峰都增大。为获得相同的谱宽和谱峰, 图 2a 需要比图 2b 更大的功率  $P_1$  和  $P_2$ 。

当光波波长处于光纤零色散波长附近时, 可忽略二阶色散的影响, 则只有四阶色散对调制不稳定增益谱有贡献。研究表明, 此时的增益谱构成及随入纤功

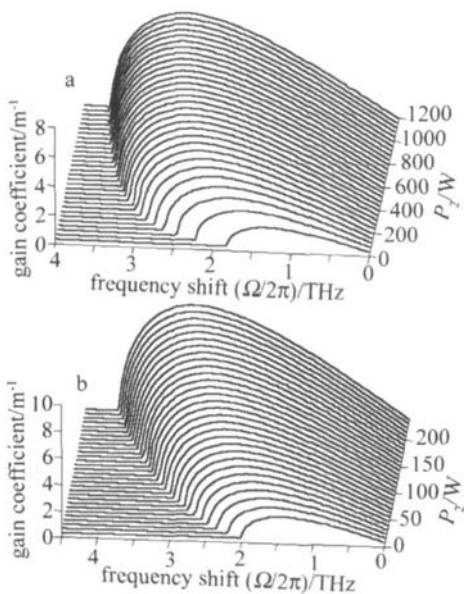


Fig. 2 Variation of gain spectra with the incident optical power  $P_2$  when the second order and the fourth order dispersion coefficients have opposite signs

率  $P_2$  的变化规律与图 2 的情形类似,且正四阶色散区比负四阶色散区具有更高更宽的增益谱,故不再赘述。

以上增益谱特征的出现可从不稳定条件  $f_1 f_2 < C_{\text{XIM}}$  的深入分析得到较好解释。由(3)式和(4)式可推知不稳定性条件为:

$$(\Omega^2 - \Omega_1^2)(\Omega^2 - \Omega_2^2)(\Omega^2 - \Omega_3^2)(\Omega^2 - \Omega_4^2) < 0 \quad (6)$$

式中:  $\Omega_{1,2,3,4}^2 = \{6\beta_2 \pm [36\beta_2^2 + 24\beta_4 Y(P_1 + P_2) \pm 24\beta_4 Y((P_1 + P_2)^2 + 12P_1P_2)^{1/2}\}]^{1/2}\}/\beta_4$  (7)

式中,前、后都取“+”时为  $\Omega_1^2$ ,前“+”后“-”时为  $\Omega_2^2$ ,前“-”后“+”时为  $\Omega_3^2$ ,前、后都取“-”时为  $\Omega_4^2$ 。

由上两式可知,当  $\beta_2 > 0$   $\beta_4 > 0$  ( $\beta_2 < 0$   $\beta_4 < 0$ ) 时,必有  $\Omega_1^2 > 0$   $\Omega_3^2 < 0$  ( $\Omega_4^2 > 0$   $\Omega_2^2 < 0$ ),若参数进一步满足不等式:

$$3\beta_2^2 - 2|\beta_4|Y\{\sqrt{(P_1 + P_2)^2 + 12P_1P_2} - |\text{sgn}(\beta_4)|(P_1 + P_2)\} > 0 \quad (8)$$

则有  $\Omega_2^2 > 0$   $\Omega_4^2 > 0$   $\Omega_4^2 < \Omega_2^2 < \Omega_1^2$  ( $\Omega_1^2 > 0$   $\Omega_3^2 > 0$   $\Omega_1^2 < \Omega_3^2 < \Omega_4^2$ ) 成立,此时,若扰动频率满足  $0 < |\Omega| < |\Omega_4|$  或  $|\Omega_2| < |\Omega| < |\Omega_1|$  ( $0 < |\Omega| < |\Omega_1|$  或  $|\Omega_3| < |\Omega| < |\Omega_4|$ ),则不稳定性产生,从而出现图 1 中  $P_2$  较小时那样的两个分离谱区,随着  $P_2$  的增大,  $|\Omega_1|$ ,  $|\Omega_4|$  增大,而  $|\Omega_2|$  ( $|\Omega_3|$ ) 减小,故两谱区都变宽,间距变小。当  $P_2$  增大到使(8)式反号时,必有  $(\Omega^2 - \Omega_2^2)(\Omega^2 - \Omega_4^2) > 0$  ( $(\Omega^2 - \Omega_1^2)(\Omega^2 - \Omega_3^2) > 0$ ) 成立,而  $\Omega_1^2 > 0$   $\Omega_3^2 < 0$  ( $\Omega_4^2 > 0$   $\Omega_2^2 < 0$ ) 结论不变,则此时的调制不稳定将只有  $0 < |\Omega| < |\Omega_1|$  ( $0 < |\Omega| < |\Omega_4|$ ) 一个谱区,如图 1 中  $P_2$  较大时所示的那样。

当  $\beta_2 > 0$   $\beta_4 < 0$  ( $\beta_2 < 0$   $\beta_4 > 0$ ) 时,必有  $\Omega_4^2 > 0$   $\Omega_2^2 < 0$  ( $\Omega_1^2 > 0$   $\Omega_3^2 < 0$  ( $\Omega_2^2 < 0$   $\Omega_4^2 < 0$ )) 成立,此时调制不稳定将只有  $0 < |\Omega| < |\Omega_4|$  ( $0 < |\Omega| < |\Omega_1|$ ) 一个谱区,当  $P_2$  增大到使(8)式反号时,必有  $(\Omega^2 - \Omega_1^2)(\Omega^2 - \Omega_3^2) > 0$  ( $(\Omega^2 - \Omega_2^2)(\Omega^2 - \Omega_4^2) > 0$ ) 成立,而  $\Omega_4^2 > 0$   $\Omega_2^2 < 0$  ( $\Omega_1^2 > 0$   $\Omega_3^2 < 0$ ) 的结论不变,调制不稳定将仍然只有  $0 < |\Omega| < |\Omega_4|$  ( $0 < |\Omega| < |\Omega_1|$ ) 一个谱区,但谱宽变宽了,正如图 2 所示的那样。

当  $\beta_2 = 0$  时,可得不稳定条件  $f_1 f_2 < C_{\text{XIM}}$  成为:

$$\Omega^2 < \Omega_c^2 =$$

$$\sqrt{\frac{1}{2} \{ \sqrt{(\Omega_{c1}^4 + \Omega_{c2}^4)^2 + 12\Omega_{c1}^4\Omega_{c2}^4} + [\text{sgn}(\beta_4)] (\Omega_{c1}^4 + \Omega_{c2}^4) \}} \quad (9)$$

式中,参数  $\Omega_j^4 = 48P_j/\beta_4$  ( $j = 1, 2$ )。

可见,当  $\beta_2 = 0$  时,不稳定性只出现  $0 < |\Omega| < |\Omega_c|$  一个谱区,且  $\beta_4 < 0$  时的谱宽较  $\beta_4 > 0$  时的更宽,与计算的谱图是一致的。

### 3 结论

在考虑到光纤的二阶到四阶色散时,分二阶、四阶色散系数同号、异号和二阶色散系数为 0 几种情形,计算了双光束的交叉相位调制不稳定性增益谱随入纤光功率的变化规律,并对出现各种谱特性的机制作了详细分析。结果表明,交叉相位调制不稳定性条件和增益谱只受二阶和四阶色散的影响,而不受三阶色散的影响。当二阶、四阶色散系数同号时,随着入纤功率由小变大,增益谱由开始的两个分离谱区逐渐变宽并合成一个谱区。当二阶、四阶色散系数异号和二阶色散系数为 0 时,增益谱只有靠近零点的第 1 谱区,且谱宽和谱峰随着入纤功率的增大而增大。对谱机制的解析分析与计算结果是一致的。

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计算1.444μm的激光输出,计算结果如图5所示。

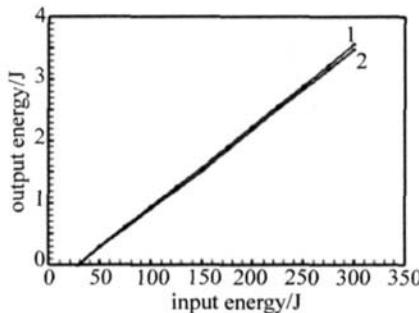


Fig. 5 The ASE effect to the laser output 1—without considering ASE effect 2—considering ASE effect

图5中曲线1和曲线2分别为不考虑ASE和考虑ASE影响的1.444μm激光输出。可以看出,随着抽运功率密度增加,ASE影响增大。在晶体棒侧面及端面反射率为0的理想条件下,ASE对功率输出的影响较小,在输入能量为300J的情况下约为2%。但如果在晶体棒侧面及端面反射率较高,ASE会急剧增大,所以激光器设计时,不应完全忽略ASE的影响。

### 3 小结

通过对其它谱线产生激光振荡条件的分析和激光输出的模拟,得出Nd:YAG激光器输出1.444μm激光的关键在于对1.064μm谱线振荡的抑制。脉冲抽运情况下,1.064μm谱线完全不产生激光振荡的反射镜临界反射率很小,但在一定范围内1.064μm谱线的激

光振荡对1.444μm激光输出的影响不是很大。通过ASE的简单模型分析了ASE对1.444μm激光输出的影响,在晶体棒侧面及端面通过处理,反射率为0的理想条件下,影响很小,可以忽略。

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