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复宗量拉盖尔-高斯光束通过矩形光阑的传输

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摘要: 利用拉盖尔-高斯模和厄米-高斯模的变换公式,并使用将二维矩孔光阑窗口函数展开为有限个复高斯函数之和的方法,得到了复宗量拉盖尔-高斯(LG)光束通过含矩形硬边光阑近轴 *ABCD* 光学系统的变换的解析传输公式,并对复宗量 LG 光束通过矩形硬边光阑的衍射和聚焦特性进行了研究。该方法对研究有不同几何对称性光阑光学系统的光束变换是有用的。

关键词: 激光光学; 复宗量拉盖尔-高斯(LG)光束; 矩形硬边光阑; 复高斯函数
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Propagation of complex-argument Laguerre-Gaussian beams through a rectangular hard-edged aperture

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Abstract Using the relations between Hermite and Laguerre-Gaussian (LG) modes and expanding the window function of two-dimensional rectangular hard-edged apertures into a finite sum of complex Gaussian functions, the propagation of complex-argument Laguerre-Gaussian beams through a paraxial optical *ABCD* system with a rectangular hard-edged aperture is studied. An analytical propagation equation is derived and used to study the diffraction at the rectangular hard-edged aperture and the focusing properties of complex-argument LG beams. The method is useful for studying the beam transformation through an apertured optical system with different geometrical symmetry.

Key words laser optics; complex-argument Laguerre-Gaussian (LG) beam; rectangular hard-edged aperture; complex Gaussian function

引 言

实际的光束在不同程度上会受到光阑的限制,对各种光束通过硬边光阑光学系统的传输变换问题已进行了许多研究。但迄今为止多限于旋转对称圆或椭圆形光束通过圆孔(或椭圆)光阑,或轴对称矩形光束通过矩孔光阑的问题[1~5]。实际工作中有时需要将矩形截面光束通过圆孔光阑后变为旋转对称光束,或处理相反的问题。作者以旋转对称复宗量拉盖尔-高斯(LG)光束通过含矩形硬边光阑近轴 *ABCD* 光学系统的变换为例,利用拉盖尔-高斯模和厄米-高斯模的变换公式,并使用将二维矩孔光阑窗口函数展开为有限个复高斯函数之和的方法,得到了解析的传输公式,还用数值计算例说明方法的应用。

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1 计算公式

初始场分布 $\Psi_0(x_0, y_0, 0)$ 的激光束通过 x, y 方向半宽分别为 a, b 的矩形硬边光阑 *ABCD* 光学系统后,场分布由 Collins 公式给出[6]:

$$\Psi(x, y, z) = \left(\frac{\mathrm{i}}{\lambda B}\right)\exp(\mathrm{i}kz)\int_{-a}^{a}\int_{-b}^{b}\Psi_0(x_0, y_0, 0)\times \exp\left\{-\frac{\mathrm{i}k}{2B}\left[A(x_0^2+y_0^2)+D(x^2+y^2)-2(x_0x+y_0y)\right]\right\}\times \mathrm{d}x_0\,\mathrm{d}y_0 \tag{1}$$

式中,k 为波数,与波长 λ 的关系为 $k=2\pi/\lambda$。硬边光阑的窗口函数可表示为二维矩形函数:

$$\mathrm{rect}(x, y) = \begin{cases} 1 & |x| \leqslant a, \ |y| \leqslant b \\ 0 & |x| > a, \ |y| > b \end{cases} \tag{2}$$

为了得到光束通过硬边光阑的变换解析公式,使用 WEN 的方法[2],将矩形函数展开为 10 项复高斯函数的叠加:

$$f(x, y) = \sum_{j=1}^{10} F_j \exp\left(-\frac{G_j}{a^2}x^2\right)\sum_{l=1}^{10} F_l \exp\left(-\frac{G_l}{b^2}y^2\right) \tag{3}$$

式中, F_j, G_j 的取值见文献[2]中表 1。将(3)式代入(1)式得:

$$\Psi(x,y,z)=\left(\frac{\mathrm{i}}{\lambda B}\right)\exp(\mathrm{i}kz)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi_0(x_0,y_0,0)\times f(x_0,y_0)\exp\{(-\mathrm{i}k/2B)[A(x_0^2+y_0^2)+D(x^2+y^2)-2(x_0x+y_0y)]\}\mathrm{d}x_0\mathrm{d}y_0 \quad (4)$$

在 $z=0$面处复宗量 LG 光束场分布为[7]:

$$\Psi_{mn}^{(0)}(\rho,\theta,0)=(-1)^n n!\rho^m\mathrm{L}_n^m(\rho^2)\mathrm{e}^{-\rho^2}\mathrm{e}^{\mathrm{i}m\theta} \quad (5)$$

式中, $\rho=r/w_0$; $m=0,\pm1,\cdots$; $n=0,1,\cdots$。

直接求(5)式表征的复宗量 LG 光束通过矩形光阑的传输遇到困难, 为此利用 LG 光束模变换为厄米-高斯模的公式[8]:

$$\mathrm{e}^{\mathrm{i}n\theta}\rho^m\mathrm{L}_n^m(\rho^2)=\frac{(-1)^n}{2^{2n+m}n!}\sum_{r=0}^{n}\sum_{s=0}^{m}\mathrm{i}^s\times\begin{bmatrix}n\\r\end{bmatrix}\begin{bmatrix}m\\s\end{bmatrix}\mathrm{H}_{2r+m-s}(x)\mathrm{H}_{2n-2r+s}(y) \quad (6)$$

将(5)式带入(4)式, 并用(3)式和(6)式得到:

$$\Psi_{mn}^{(0)}(x,y,z)=\left(\frac{\mathrm{i}}{\lambda B}\right)\exp(\mathrm{i}kz)\exp\left[-\frac{\mathrm{i}kD}{2B}(x^2+y^2)\right]\times\frac{1}{2^{2n+m}}\sum_{r=0}^{n}\sum_{s=0}^{m}\mathrm{i}^s\begin{bmatrix}n\\r\end{bmatrix}\begin{bmatrix}m\\s\end{bmatrix}\sum_{j=1}^{10}\int_{-\infty}^{+\infty}F_j\exp\left(-\frac{x_0^2}{w_0^2}\right)\times\exp\left(-\frac{G_jx_0^2}{a^2}\right)\exp\left[-\frac{\mathrm{i}k}{2B}(Ax_0^2-2x_0x)\right]\mathrm{H}_{2r+m-s}(x_0)\mathrm{d}x_0\times\sum_{l=1}^{10}\int_{-\infty}^{+\infty}F_l\exp\left(-\frac{y_0^2}{w_0^2}\right)\exp\left(-\frac{G_ly_0^2}{a^2}\right)\times\exp\left[-\frac{\mathrm{i}k}{2B}(Ay_0^2-2y_0y)\right]\mathrm{H}_{2n-2r+s}(y_0)\mathrm{d}y_0 \quad (7)$$

利用积分公式:

$$\int_{-\infty}^{+\infty}\exp\left[-\frac{(x-y)^2}{2u}\right]\mathrm{H}_n(x)\mathrm{d}x=\sqrt{2\pi u}(1-2u)^{n/2}\mathrm{H}_n\left[\frac{y}{(1-2u)^{1/2}}\right] \quad (8)$$

积分(7)式, 得:

$$\Psi_{nm}^{(0)}(x,y,z)=\left(\frac{\mathrm{i}k}{2B}\right)\exp(\mathrm{i}kz)\exp\left[-\frac{\mathrm{i}kD}{2B}(x^2+y^2)\right]\times\frac{1}{2^{2n+m}}\sum_{r=0}^{n}\sum_{s=0}^{m}\mathrm{i}^s\begin{bmatrix}n\\r\end{bmatrix}\begin{bmatrix}m\\s\end{bmatrix}\sum_{j=1}^{10}\frac{F_j}{Q_j}\left(1-\frac{2}{Q_j^2}\right)^{\frac{2r+m-s}{2}}\times\exp\left[-\left(\frac{k}{2BQ_j}\right)^2x^2\right]\mathrm{H}_{2r+m-s}[R_jx]\sum_{l=1}^{10}\frac{F_l}{Q_l}\left(1-\frac{2}{Q_l^2}\right)^{\frac{2n-2r+s}{2}}\times\exp\left[-\left(\frac{k}{2BQ_l}\right)^2y^2\right]\mathrm{H}_{2n-2r+s}[R_ly] \quad (9)$$

式中,

$$Q_j^2=\frac{1}{w_0^2}+\mathrm{i}k\frac{A}{B}+\frac{G_j}{a^2},\ R_j=\frac{\mathrm{i}k}{2BQ_j(Q_j^2-1)^{1/2}},$$
$$Q_l^2=\frac{1}{w_0^2}+\mathrm{i}k\frac{A}{B}+\frac{G_l}{b^2},\ R_l=\frac{\mathrm{i}k}{2BQ_l(Q_l^2-1)^{1/2}} \quad (10)$$

(9)式为本文中所得主要的解析结果。从(9)式可以看出, x方向和 y方向的变量是可分离的, 径向和角向指数分别为 n, m 的复宗量 LG 光束 $\Psi_{mn}^{(0)}(\rho,\theta,z)$可由 x方向为 $2r+m-s$阶和 y方向 $2n-2r+s$阶的厄米-高斯模叠加得到。

对复宗量 LG 光束经过无光阑($a\to\infty$, $b\to\infty$)的 $ABCD$ 近轴光学系统的传输由 Collins公式推出为:

$$\Psi_{mn}^{(0)}(\rho,\theta,z)=(-1)^n n!\left(\frac{\mathrm{i}}{\lambda B}\right)\mathrm{e}^{-\mathrm{i}n\theta}2\pi\mathrm{i}^m\exp(\mathrm{i}kz)\times\exp\left(-\frac{\mathrm{i}kDw_0^2}{2B}\rho^2\right)\frac{1}{2^{m+1}\left(\frac{1}{w_0^2}+\frac{\mathrm{i}kA}{2B}\right)^{m+n+1}}\left(\frac{\mathrm{i}kA}{2B}\right)^n\left(\frac{k\rho}{B}\right)^m\times\exp\left[-\frac{k^2w_0^2\rho^2}{4B^2\left(\frac{1}{w_0^2}+\frac{\mathrm{i}kA}{2B}\right)}\right]\mathrm{L}_n^m\left[-\frac{k^2\rho^2/B^2}{4\left(\frac{1}{w_0^2}+\frac{\mathrm{i}kA}{2B}\right)\frac{\mathrm{i}kA}{2B}}\right] \quad (11)$$

2　数值计算与分析

在数值计算中以 TEM_{10}为例($m=1$, $n=0$), 瑞利长度 $Z_0=\pi w_0^2/\lambda$, 截断参数 $\delta=a/w_0$。取 $f=100\mathrm{mm}$, $w_0=1\mathrm{mm}$, $\lambda=1.06\mu\mathrm{m}$。图 1中给出了复宗量 LG 光束

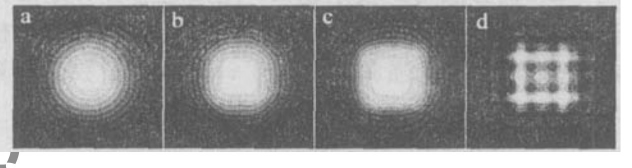

Fig. 1　Contour lines of the intensity of a complex-argument TEM_{10} mode LG beam passing through a rectangular hard-edged aperture-lens system
a—$\delta=3.0$　b—$\delta=1.5$　c—$\delta=1.0$　d—$\delta=0.5$

通过在 $z=0$平面上置一方孔($a=b$)硬边光阑薄透镜后在 $z=2f$面处用(9)式通过数值计算得到的横向光强分布等高线图, $ABCD$ 矩阵的矩阵元分别为 $A=-(z-f)/f=-\Delta z$, $B=f(1+\Delta z)$, $C=-1/f$, 截断参数为 $\delta=3.0$, 1.5, 1.0, 0.5。从图 1可以看出, 随着截断比的减小, 衍射效应增强, 对称性发生变化。与用(11)式作数值计算比较可知, 当 $\delta\geqslant3.0$时二者所得结果一致, 即 $\delta\geqslant3.0$时光阑衍射效应可以忽略。图 2是复宗量 LG光束通过方孔硬边光阑薄透镜光学系统

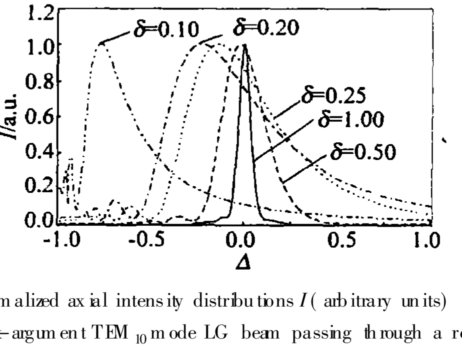

Fig. 2　Normalized axial intensity distributions I(arbitrary units) of a complex-argument TEM_{10} mode LG beam passing through a rectangular hard-edged aperture-lens system

后的轴上归一化光强分布, 由图 2知, 此时轴上光强最大点位置不在几何焦面 $\Delta z=0$处, 而向光阑透镜方向移动, 即出现焦移。相对焦移 $\Delta=(z_{\max}-f)/f$($z_{\max}$为最

大光强位置)与截断参数有关, 当截断参数 δ= 1.00, 0.50, 0.30, 0.25和 0.10时, 相对焦移 Δ分别为: −0.01, −0.02, −0.14, −0.23, −0.77, 因此|Δ|随 δ增加而减小。图 3是相对焦移 Δ随菲涅耳数 $N = a^2/\lambda f$的变化, 由图 3可知, |Δ|随 N 增加而减小, 当菲涅耳数 $N = 3$, 相对焦移|Δ|≈ 0。

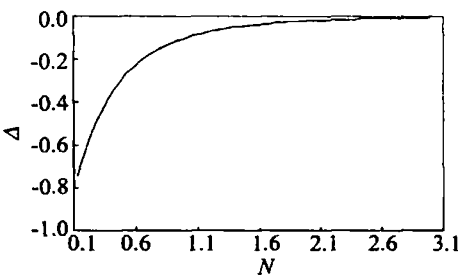

Fig. 3 Relative focal shift Δ versus Fresnel number N

3 结 论

利用拉盖尔-高斯模和厄米-高斯模间的变换关系将复宗量拉盖尔-高斯模转换为复宗量厄米-高斯模, 用 WEN的方法得到复宗量 LG光束通过含有矩形硬边光阑近轴 $ABCD$ 光学系统的解析计算公式, 对其横向场分布、轴上光强和焦移作了计算分析。数值计算表明, TEM_{10}模复宗量LG光束通过光阑透镜后、当 δ≥3.0时横向光强分布与无光阑时的解析解所得结果一致。作者所用方法为旋转对称光束通过轴对称矩形硬边光阑光学系统变换的研究提供了一种可行的研究方法, 具有实际应用意义。

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superfluorescent fiber source can be done in SFS wavelength stability, conversion efficiency, output power, and compactness. Firstly, high quality Er^{3+}/Yb^{3+} co-doped fiber with lower loss, higher conversion efficiency and optimum fiber dimensions should be chosen. Secondly, a new pump source well-matched the fiber if it is possible, may replace the one used in this experiment. Lastly, integration of the superfluorescent fiber source should be considered in order to make the system more compact and stable.

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