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基于 SPIDER法测量飞秒脉冲的数值计算与模拟

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摘要: 为了测量飞秒脉冲的振幅和相位,采用光谱相位相干电场重构法(SPIDER)的光谱图数值分析公式,对高斯强度分布的线性啁啾脉冲、立方相位啁啾脉冲进行数值计算与数值模拟,并重构出飞秒脉冲的光谱相位,其模拟的结果与理论符合较好,光谱相位相干电场重构法能完全测量飞秒脉冲的振幅和相位。

关键词: 超快光学;飞秒脉冲测量;光谱相位相干电场重构法;数值模拟
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Numerical simulation of femtosecond pulses measurement with SPIDER

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Abstract: The object is to measure the amplitude and phase of femtosecond pulses, based on the spectral phase interferometry for direct electric-field reconstruction (SPIDER) of the numerical analysis methods. The spectral phases and intensity of the typical femtosecond pulses were numerical calculated, the spectral phases of these pulses are reconstructed. The reconstructed spectral phases are basically in agreement with the original ones.

Key words: ultrafast optics; femtosecond pulses measurement; spectral phase interferometry for direct electric-field reconstruction (SPIDER); numerical simulation

引　言

目前,国际上公认的用于飞秒脉冲测量的方法主要是基于自相关法的频率分辨光学开关法(FROG)和自参考光谱相位相干电场重构法(SPIDER)。FROG和 SPIDER测量方法的优点是都可以直接从测量所得数据中构建出飞秒脉冲的振幅和相位等信息。但 FROG的反演算法复杂且速度较慢; SPIDER的反演算法简单而快捷,因此可作为实时测量的工具[1~3]。SPIDER是基于一种叫做光谱剪裁相干的方法,它是将待测脉冲经分束器分为两束,其中一束通过一色散介质以展宽啁啾脉冲,另一束经迈克尔逊干涉仪分为 2个具有时间延迟的一对脉冲,然后再将它们聚焦到倍频晶体中进行频率转换,和频后的脉冲对经过光谱仪记录下光谱相干的干涉条纹,再利用反演算法,就可得到入射脉冲的相位。

作者采用 SPIDER的光谱图数值分析公式,对高斯强度分布的线性啁啾脉冲、立方相位啁啾脉冲进行了数值计算及模拟,重构光谱相位。

1　光谱相位重构的数值分析

1.1　干涉光谱信号表达式

采用 SPIDER法测量飞秒脉冲的装置如图 1所示。

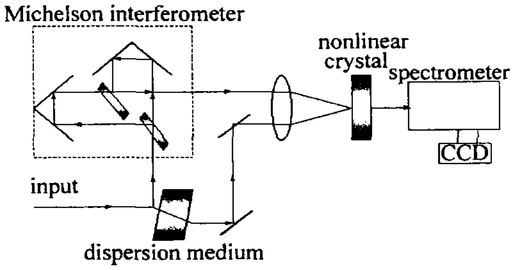

Fig. 1　Schematic of the SPIDER

用光谱仪可测得两束光通过倍频晶体的干涉光谱信号,标准的干涉光谱信号表达式[4]:

$$D(\omega_c) = |\tilde{E}(\omega_c - \Omega)|^2 + |\tilde{E}(\omega_c)|^2 + 2|\tilde{E}(\omega_c - \Omega)||\tilde{E}(\omega_c)|\cos[\phi(\omega_c - \Omega) - \phi(\omega_c) - \tau\omega_c] \quad (1)$$

光谱相位相干直接电场重构法就是要从(1)式中提取相位差 $\phi(\omega_c - \Omega) - \phi(\omega_c)$,进而重构光谱相位 $\phi(\omega_c)$。式中,ω_c是中心载波频率,Ω是光谱剪裁量,

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$\Omega = -\tau/\phi''$,时间延迟 τ由迈克尔逊干涉仪测量得到, ϕ''是倍频晶体的二阶色散, ϕ''可通过简单计算得到。

1.2 对光谱数据的反演运算重构光谱相位

对光谱干涉图数据的傅里叶变换、滤波和逆变换运算后,提取了 (1)式的相位差,得到经过频移后的脉冲与没有频移的脉冲之间的相位差:

$$\theta(\omega_c) = \phi(\omega_c - \Omega) - \phi(\omega_c) \tag{2}$$

而光谱相位通过积分能重构[4]:

$$\phi(\omega_c) \approx \frac{1}{\Omega}\int \theta(\omega_c)\,\mathrm{d}\omega_c \tag{3}$$

2 光谱相位重构的数值模拟

在数值模拟中,以高斯强度分布的脉冲为入射脉冲,其宽度 $t_{in} = 40\mathrm{fs}$,中心载波频率 $\omega_c = 375\mathrm{THz}$(中心波长 $\lambda = 800\mathrm{nm}$),经迈克尔逊干涉仪后产生两脉冲时间延迟 $\tau = 2.1\mathrm{ps}$,展宽器由相距为 10cm的一对平行光栅 ($1/d = 1200$条 /mm)组成,提供的色散量[5]为 $\phi'' = 3.33 \times 10^5\,\mathrm{fs}^2$,计算得到的频率剪裁量 $\Omega = 2.01\pi\,\mathrm{THz}$。

2.1 入射线性啁啾脉冲的数值模拟

线性啁啾脉冲:

$$E(t) = \exp(-0.001t^2 + \mathrm{i}0.001t^2) \cdot \exp(-\mathrm{i}\omega_c t) \tag{4}$$

则:

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t = \int_{-\infty}^{\infty} \exp(-0.001t^2 + \mathrm{i}0.001t^2 - \mathrm{i}\omega_c t + \mathrm{i}\omega t)\,\mathrm{d}t = 47.13\mathrm{e}^{-125(\omega-\omega_c)^2} \times \exp\{\mathrm{i}[0.3925 - 125(\omega - \omega_c)^2]\} \tag{5}$$

振幅谱为:

$$|\tilde{E}(\omega)| = |\tilde{E}(\omega_c)| = 47.13\mathrm{e}^{-125(\omega-\omega_c)^2} \tag{6}$$

相位谱为:

$$\phi(\omega) = \phi(\omega_c) = 0.3925 - 125(\omega - \omega_c)^2 \tag{7}$$

干涉光谱强度:

$$D(\omega_c) = |\tilde{E}(\omega_c - \Omega)|^2 + |\tilde{E}(\omega_c)|^2 + 2|\tilde{E}(\omega_c - \Omega)| \times |\tilde{E}(\omega_c)| \cos[\phi(\omega_c - \Omega) - \phi(\omega_c) - \tau\omega_c] = 47.13^2 \times \exp[-250(\omega - \omega_c + \Omega)^2] + 47.13^2 \exp[-250(\omega - \omega_c)^2] + 2 \times 47.13^2 \exp[-125(\omega - \omega_c + \Omega)^2] \cdot \exp[-125(\omega - \omega_c)^2] \cos[-125\Omega^2 - 250\Omega(\omega - \omega_c) - \tau\omega_c] \tag{8}$$

光谱相位通过积分重构:

$$\phi(\omega_c) \approx \frac{1}{\Omega}\int \theta(\omega_c)\,\mathrm{d}\omega_c = \frac{1}{\Omega}\int [\phi(\omega_c - \Omega) - \phi(\omega_c)]\,\mathrm{d}\omega_c = -125\int(\Omega + 2\omega - 2\omega_c)\,\mathrm{d}\omega_c = -125(\omega - \omega_c + \Omega/2)^2 + C \tag{9}$$

由数值模拟得到的入射脉冲的光谱强度和相对相位分布见图 2,重构的光谱相位见图 3,对光谱强度和重构的光谱相位再做一次傅里叶变换,就还原出入射的线性啁啾脉冲在时域的强度和相位,见图 3中的插图。

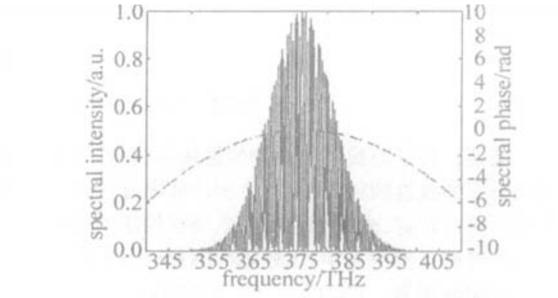

Fig 2 Spectral intensity and phase for the linearly chirped pulse, solid curve—the intensity, dotted curve—the phase

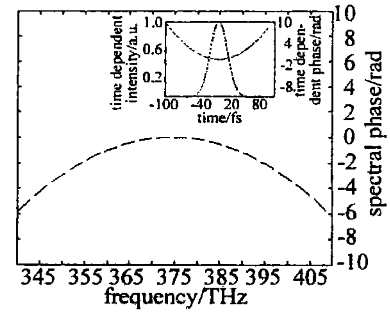

Fig 3 Reconstructed spectral phase of the linearly chirped pulse, upper right—reconstructed time dependent intensity profile and phase of the pulse

2.2 入射立方相位啁啾脉冲的数值模拟

立方相位啁啾脉冲:

$$E(t) = \exp(-0.001t^2 + \mathrm{i}0.00001t^3) \cdot \exp(-\mathrm{i}\omega_c t) \tag{10}$$

则:

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} \exp(-0.001t^2 + \mathrm{i}0.00001t^3 - \mathrm{i}\omega_c t + \mathrm{i}\omega t)\,\mathrm{d}t \approx 63.24\{0.59 + 0.89\cos[0.113 + 22.5(\omega - \omega_c)] + 0.30\cos[0.574 + 38.6(\omega - \omega_c)] + 0.80\cos[0.0445 + 16.4(\omega - \omega_c)] + 0.48 + 0.39\cos[0.280 + 30.4(\omega - \omega_c)] + 0.72\cos[0.0262 + 13.8(\omega - \omega_c)] + 0.16\cos[0.754 + 42.2(\omega - \omega_c)] + 0.43\cos[0.172 + 25.8(\omega - \omega_c)] + 0.66\cos[0.0173 + 12.0(\omega - \omega_c)] + 0.21\cos[0.493 + 36.7(\omega - \omega_c)] + 0.43\cos[0.118 + 22.8(\omega - \omega_c)] + 0.61\cos[0.0127 + 10.8(\omega - \omega_c)] + 0.24\cos[0.356 + 32.9(\omega - \omega_c)]\} \tag{11}$$

式中, $\tilde{E}(\omega)$的相位数学表达式无法求解出,但 MATLAB提供了复数函数相角的可视化功能[6],从图 4可知,相位表达式为:

$$\phi(\omega) = \phi(\omega_c) = \begin{cases} 3.2, & 331 < \omega/2\pi < 361 \\ 0, & 361 < \omega/2\pi < 389 \\ 3.2, & 389 < \omega/2\pi < 419 \end{cases} \tag{12}$$

$\theta(\omega_c) = \phi(\omega_c - \Omega) - \phi(\omega_c) \approx 0.069\sin[0.0172(\omega -$

$$\omega_c)] + 0.0075\sin[0.0345(\omega - \omega_c)] - 0.068 \times \sin[0.0517(\omega - \omega_c)] - 0.015\sin[0.0690(\omega - \omega_c)] + 0.065\sin[0.0862(\omega - \omega_c)] + 0.022\sin[0.103(\omega - \omega_c)] - 0.062\sin[0.120(\omega - \omega_c)] - 0.028 \times \sin[0.138(\omega - \omega_c)] \qquad (13)$$

干涉光谱强度：

$$D(\omega_c) = |\tilde{E}(\omega_c - \Omega)|^2 + |\tilde{E}(\omega_c)|^2 + 2|\tilde{E}(\omega_c - \Omega)| \times |\tilde{E}(\omega_c)| \cos[\phi(\omega_c - \Omega) - \phi(\omega_c) - \tau\omega_c] \approx 2|\tilde{E}(\omega_c)|^2\{1 + \cos[\phi(\omega_c - \Omega) - \phi(\omega_c) - \tau\omega_c]\} \quad (\text{因 } \Omega \text{ 较小}) \qquad (14)$$

干涉光谱强度包络：$D(\omega) \propto \tilde{E}(\omega_c)^2$　　(15)

光谱相位通过积分重构：

$$\phi(\omega_c) \approx \frac{1}{\Omega}\int\theta(\omega_c)\,d\omega_c = -4.0\cos[0.0172(\omega - \omega_c)] - 0.22\cos[0.0345(\omega - \omega_c)] + 1.3\cos[0.0517(\omega - \omega_c)] + 0.22\cos[0.0690(\omega - \omega_c)] - 0.76\cos[0.0862(\omega - \omega_c)] - 0.21\cos[0.103(\omega - \omega_c)] + 0.52\cos[0.120(\omega - \omega_c)] + 0.20\cos[0.138(\omega - \omega_c)] \qquad (16)$$

由数值模拟得到的入射脉冲的光谱强度包络和相对相位分布见图4，重构的光谱相位见图5。由于立方相位啁啾脉冲光谱强度是根据积分近似计算得出的多项式，再做一次傅里叶变换还原出入射的立方相位啁啾脉冲在时域的强度在数学上就比较困难。

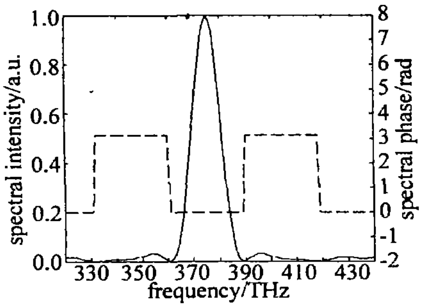

Fig 4　Spectral intensity profile and phase for the pulse with temporal cubic phase distortion, solid curve—the intensity, dotted curve—the phase

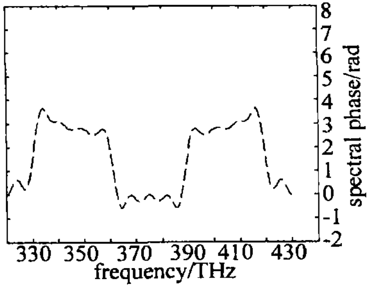

Fig 5　Reconstructed spectral phase of the pulse with temporal cubic phase distortion

3　分析和结论

图2是数值模拟的高斯型线性啁啾脉冲光谱强度和相位，频谱半宽度 $\Delta\nu = 16\text{THz}$，而入射脉冲宽度 $t_{in} = 40\text{fs}$，由此得 $\Delta t_{in} \cdot \Delta\nu = 0.64 > 0.44$[5]，说明频谱宽度依赖于啁啾而扩展。图3重构光谱相位再现了图2原有光谱相位。图4是数值模拟的高斯型立方相位啁啾脉冲光谱强度包络和相位，频谱半宽度 $\Delta\nu = 12\text{THz}$，而入射脉冲宽度 $t_{in} = 40\text{fs}$，由此得 $\Delta t_{in} \cdot \Delta\nu = 0.48 > 0.44$[5]，说明立方相位啁啾脉冲的中心部分是无啁啾，但脉冲两翼存在啁啾，频谱宽度基本上等于变换极限宽度。图5重构光谱相位较好地再现了图4原有光谱相位，立方相位曲线在时域呈S形状，在频域相位曲线呈矩形波状，这是典型的脉冲含有三阶色散的相位曲线。

采用飞秒脉冲测量的光谱相位相干电场重构法(SPIDER)的光谱图数值分析公式，对高斯强度分布的线性啁啾脉冲、立方相位啁啾脉冲进行严格的数学推导、数值计算及数值模拟，其模拟的结果与理论符合较好，光谱相位相干电场重构法能完全测量飞秒脉冲的振幅和相位。

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