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会聚球面波通过环形光阑的光强分布

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摘要: 基于菲涅耳衍射积分公式,推导出了会聚球面波通过环形光阑后场分布的解析公式,并讨论了一些特殊情况。数值计算例表明,光强分布与菲涅耳数和遮拦比有关。使用轴上光强公式和近似公式对焦移计算结果的比较证明了近似公式的适用范围。

关键词: 会聚球面波;环形光阑;光强分布;焦移;有效菲涅耳数
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Intensity distribution of converging spherical waves passing through an annular aperture

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Abstract: Based on the Fresnel diffraction integral, the analytical expression for the field distribution of converging spherical waves passing through an annular aperture is derived and some special cases are discussed. Numerical calculations are performed to show the dependence of intensity distribution on the Fresenl number and obscure ratio. In order to illustrate the valid range of the axial intensity expression and approximation formulae numerical results of both the methods are compared.

Key words: converging spherical wave; annular aperture; intensity distribution; focal shift; effective Fresnel number

引　言

由非稳腔输出的环形光束可等价为通过一个环形光阑的球面波(特殊情况为环形平面波)。对会聚球面波通过环形光阑的传输变换问题,已有很多文献报道,其中,重点是轴上光强分布和焦移[1~3]。笔者的主要目的是:(1)在近轴近似下,推导出会聚球面波通过环形光阑后场分布的解析公式,并作计算分析;(2)对与场(或光强)分布直接相关的计算焦移的公式作分析比较,得出了近似公式的适用范围。

1　会聚球面波通过环形光阑场分布的解析公式

如图 1所示,一单色均匀会聚球面波照射一内外半径分别为 εa $(0<\varepsilon<1)$和 a的环形光阑(ε为遮拦比),O为光阑中心,F为会聚球面波的几何焦点,$f=OF$。

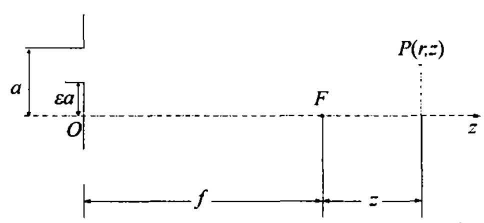

Fig 1　Propagation of converging spherical waves passing through an annular aperture

当　$a \gg \lambda,\ (a/f)^2 \ll 1$　(1)

(λ为波长)时,可将光阑面上场分布表示为[4]:

$$E_0(r_1) = \frac{\exp(-ikf)}{f}\exp\left(-\frac{ikr_1^2}{2f}\right) \tag{2}$$

式中,$k=2\pi/\lambda$为波数。光阑后 $P(r,z)$点处场分布可由菲涅耳衍射积分公式计算:

$$E(r,z) = \frac{-iu_N\exp(ikz)}{z}\exp\left[i\frac{v_N}{2}\frac{r}{a}\right]\times \int_\varepsilon^1 \exp\left[-i\frac{u_N}{2}r'^2\right]J_0[v_N r']r'\mathrm{d}r' \tag{3}$$

式中,J_0为零阶贝塞尔函数。

$$N = a^2/\lambda f \text{(圆孔光阑的菲涅耳数)} \tag{4}$$

$$u_N = 2\pi N\frac{z/f}{(1+z/f)} \tag{5a}$$

$$v_N = 2\pi N\frac{r/a}{(1+z/f)} \tag{5b}$$

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利用贝塞尔函数公式:

$$\begin{cases}\dfrac{\mathrm{d}}{\mathrm{d}x}[x^n\mathrm{J}_n(x)]=x^n\mathrm{J}_{n-1}(x)\\ \exp\left[\dfrac{x}{2}\left(t-\dfrac{1}{t}\right)\right]=\sum\limits_{n=-\infty}^{+\infty}t^n\mathrm{J}_n(x)\end{cases}\tag{6}$$

对 (3)式作冗长但直接的积分运算,最后结果可整理为:

$$E(r,z)=-\frac{\mathrm{i}\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v_N}{2}\frac{r}{a}\right)\exp\left(-\frac{\mathrm{i}u_N}{2}\right)\times\left\{[U_1(u_N,v_N)+\mathrm{i}U_2(u_N,v_N)]-\exp\left[\frac{\mathrm{i}u_N(1-\varepsilon^2)}{2}\right]\times[U_1{}'(u_N,v_N)+\mathrm{i}U_2{}'(u_N,v_N)]\right\},\quad\left|\frac{u_N}{v_N}\right|<1\tag{7a}$$

$$E(r,z)=-\frac{\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v_N}{2}\frac{r}{a}\right)\left\{\exp\left(\frac{\mathrm{i}v_N^2}{2u_N}\right)-\exp\left(-\frac{\mathrm{i}u_N}{2}\right)[V_1(u_N,v_N)+\mathrm{i}V_2(u_N,v_N)]+\mathrm{i}\exp\left[\frac{\mathrm{i}u_N(1-\varepsilon^2)}{2}\right][U_1{}'(u_N,v_N)+\mathrm{i}U_2{}'(u_N,v_N)]\right\},\quad 1<\left|\frac{u_N}{v_N}\right|<\frac{1}{\varepsilon}\tag{7b}$$

$$E(r,z)=\frac{\exp(\mathrm{i}kz)}{z}\exp\left(\frac{\mathrm{i}v_N}{2}\frac{r}{a}\right)\exp\left(-\frac{\mathrm{i}u_N}{2}\right)\times\left\{[V_1(u_N,v_N)+\mathrm{i}V_2(u_N,v_N)]-\exp\left[\frac{\mathrm{i}u_N(1-\varepsilon^2)}{2}\right]\times[V_1{}'(u_N,v_N)+\mathrm{i}V_2{}'(u_N,v_N)]\right\},\quad\left|\frac{u_N}{v_N}\right|>\frac{1}{\varepsilon}\tag{7c}$$

式中,$U_n, V_n, U_n{}', V_n$ 为 Lommel函数[5]:

$$U_n(u_N,v_N)=\sum_{s=0}^{\infty}(-1)^s\left(\frac{u_N}{v_N}\right)^{n+2s}\mathrm{J}_{n+2s}(v_N)\tag{8a}$$

$$V_n(u_N,v_N)=\sum_{s=0}^{\infty}(-1)^s\left(\frac{v_N}{u_N}\right)^{n+2s}\mathrm{J}_{n+2s}(v_N)\tag{8b}$$

$$U_n{}'(u_N,v_N)=\sum_{s=0}^{\infty}(-1)^s\left(\varepsilon\frac{u_N}{v_N}\right)^{n+2s}\mathrm{J}_{n+2s}(\varepsilon v_N)\tag{9a}$$

$$V_n{}'(u_N,v_N)=\sum_{s=0}^{\infty}(-1)^s\left(\frac{v_N}{\varepsilon u_N}\right)^{n+2s}\mathrm{J}_{n+2s}(\varepsilon v_N)\tag{9b}$$

J_n 为 n阶贝塞尔函数。(7)式为本文中的主要解析结果,具有较为普遍的意义。下面讨论 (7)式的几个特例。

(1) $\varepsilon\rightarrow 0$ 时,为圆孔光阑衍射。由 (7a)式和 (7b)式得:

$$E(r,z)=-\frac{\mathrm{i}\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v_N}{2}\frac{r}{a}\right)\exp\left(-\frac{\mathrm{i}u_N}{2}\right)\times[U_1(u_N,v_N)+\mathrm{i}U_2(u_N,v_N)],\quad\left|\frac{u_N}{v_N}\right|<1\tag{10a}$$

$$E(r,z)=-\frac{\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v_N}{2}\frac{r}{a}\right)\times\left\{\exp\left(\frac{\mathrm{i}v_N^2}{2u_N}\right)-\exp\left(-\frac{\mathrm{i}u_N}{2}\right)\times[V_1(u_N,v_N)+\mathrm{i}V_2(u_N,v_N)]\right\},\quad\left|\frac{u_N}{v_N}\right|>1\tag{10b}$$

(2)当 $N\gg 1$时,有[2]:

$$u_N\approx u,\quad v_N\approx v\tag{11}$$

式中,

$$u=\frac{2\pi}{\lambda}\left(\frac{a}{f}\right)^2 z\tag{12a}$$

$$v=\frac{2\pi}{\lambda}\left(\frac{a}{f}\right)^2 r\tag{12b}$$

此时,P点的场分布为:

$$E(r,z)=-\frac{\mathrm{i}\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v}{2}\frac{r}{a}\right)\exp\left(-\frac{\mathrm{i}u}{2}\right)\times\left\{[U_1(u,v)+\mathrm{i}U_2(u,v)]-\exp\left[\frac{\mathrm{i}u(1-\varepsilon^2)}{2}\right]\times[U_1{}'(u,v)+\mathrm{i}U_2{}'(u,v)]\right\},\quad\left|\frac{u}{v}\right|<1\tag{13a}$$

$$E(r,z)=-\frac{\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v}{2}\frac{r}{a}\right)\left\{\exp\left(\frac{\mathrm{i}v^2}{2u}\right)-\exp\left(-\frac{\mathrm{i}u}{2}\right)[V_1(u,v)+\mathrm{i}V_2(u,v)]+\mathrm{i}\exp\left[\frac{\mathrm{i}u(1-\varepsilon^2)}{2}\right][U_1{}'(u,v)+\mathrm{i}U_2{}'(u,v)]\right\},\quad 1<\left|\frac{u}{v}\right|<\frac{1}{\varepsilon}\tag{13b}$$

$$E(r,z)=\frac{\exp(\mathrm{i}kz)}{z}\exp\left(\mathrm{i}\frac{v}{2}\frac{r}{a}\right)\exp\left(-\frac{\mathrm{i}u}{2}\right)\times\left\{[V_1(u,v)+\mathrm{i}V_2(u,v)]-\exp\left[\frac{\mathrm{i}u(1-\varepsilon^2)}{2}\right]\times[V_1{}'(u,v)+\mathrm{i}V_2{}'(u,v)]\right\},\quad\left|\frac{u}{v}\right|>\frac{1}{\varepsilon}\tag{13c}$$

(13)式为 $N\gg 1$时环形光阑对会聚球面波的衍射公式。容易看出,此时无焦移存在。进一步,当 $\varepsilon=0$时,由 (13a)和 (13b)式得:

$$E(r,z) = -\frac{i\exp(ikz)}{z}\exp\left(i\frac{v}{2}\frac{r}{a}\right)\exp\left(-\frac{iu}{2}\right)\times$$
$$[U_1(u,v)+iU_2(u,v)],\quad \left|\frac{u}{v}\right|<1 \qquad (14a)$$

$$E(r,z) = -\frac{\exp(ikz)}{z}\exp\left(i\frac{v}{2}\frac{r}{a}\right)\times$$
$$\left\{\exp\left(\frac{iv^2}{2u}\right)-\exp\left(-\frac{iu}{2}\right)[V_1(u,v)+iV_2(u,v)]\right\},$$
$$\left|\frac{u}{v}\right|>1 \qquad (14b)$$

(10)式和 (14)式分别与文献 [1]中 (3.14)式和文献 [5]中 8.8.1节的结论相同。值得指出的是,虽然文献 [1]、[5]中使用了对波阵面积分,而本文中用菲涅耳衍射积分公式对光阑面积分的方法,但所得结果是相同的。因此,文献 [1]、[5]也都是近轴近似下的结果。对非近轴情况应当另作研究。

(3) 在 (3)式中令 $z=0$,得环形光阑衍射几何焦面上的场分布公式:

$$E(r,0) = -\frac{i\pi N}{f}\exp\left(i\pi N\frac{r^2}{a^2}\right)\times$$
$$\left[\frac{J_1(2\pi Nr/a)}{\pi Nr/a}-\frac{\varepsilon^2 J_1(2\varepsilon\pi Nr/a)}{\varepsilon\pi Nr/a}\right] \qquad (15)$$

这是熟知的结果。

(4) 在 (3)式中令 $r=0$,得环形光阑衍射时轴上场分布公式:

$$E(0,u_N) = \frac{2\pi N-u_N}{fu_N}\exp\left(-\frac{iu_N}{2}\right)\times$$
$$\left\{1-\exp\left[\frac{i(1-\varepsilon^2)u_N}{2}\right]\right\} \qquad (16)$$

2　会聚球面波的焦移

文献 [4]中使用环形光阑的有效菲涅耳数推出焦移的近似公式为:

$$\Delta z_f = -\frac{1}{\pi^2 N_{eff}^2} \qquad (17)$$

式中,
$$N_{eff} = \frac{(1-\varepsilon^2)}{\sqrt{12}}\frac{a^2}{\lambda f} \qquad (18)$$

为环形光阑的有效菲涅耳数。

KATHURIA[3]从 (16)式出发,在 $\tan x\approx x+\frac{1}{3}x^3$ 近似下推出会聚球面波经环形光阑衍射的焦移公式为:

$$\Delta z_f' = -\frac{12}{12+(1-\varepsilon^2)^2\pi^2N^2} = -\frac{1}{1+\pi^2N_{eff}^2} \qquad (19)$$

应当指出,(17)式、(19)式都是计算会聚球面波经环形光阑衍射后焦移的近似公式,它们仅在一定条件下才与直接从 (16)式出发计算轴上光强最大值位置而求出的焦移值相等。此外,由 (17)式、(19)式算的焦移也仅在 $\pi^2N_{eff}^2\gg 1$的条件下才相等。

3　数值计算和分析

利用场分布解析公式 (7)式、轴上场分布公式 (16)式,以及 (17)式、(19)式,对会聚球面波通过环形光阑的光强分布 $I=|E|^2$ 和焦移作了计算。图 2为 $N=2$, $\varepsilon=0.5$时会聚球面波通过环形光阑后的三维光强分布。图 3为会聚球面波通过环形光阑后的归一化焦面上的光强分布。从图 2、图 3可以看出,横截面上的光强为对称分布,轴上光强最大值并不位于 $z=0$ (即几何焦面),而向光阑面有一定的移动,此即焦移。并且,焦移随 N的减小和 ε的增大而增加。当 $N=70$, $\varepsilon=0, 0.5, 0.8$时,$\Delta z_f=0$;当 $N=2$, $\varepsilon=0, 0.5, 0.8$时相对焦移 Δz_f分别为 -0.19, -0.27, -0.51。

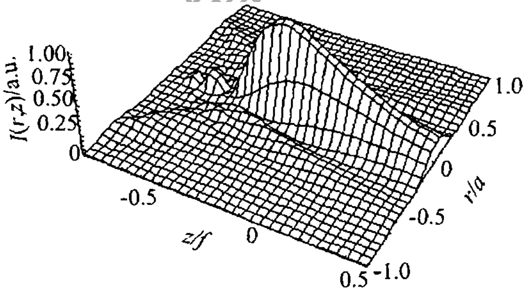

Fig 2　Normalized intensity distribution of converging spherical waves passing through an annular aperture ($N=2$, $\varepsilon=0.5$)

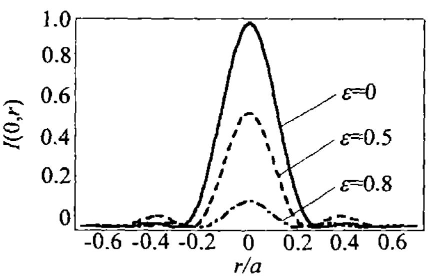

Fig 3　Normalized transversal intensity distribution of converging spherical waves passing through an annular aperture on focal plane ($N=2$)

图 4为使用轴上光强公式和近似公式 (17)式、(19)式计算的焦移随 N_{eff}的变化情况。从图中可以看

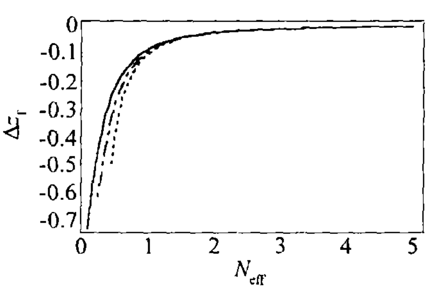

Fig 4　The relative focal shift Δz_f varies with N_{eff}　——using Eq. (16)　---—using Eq. (17)　—·——using Eq. (19)

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径。

空间载波技术在光学测量、全息干涉测量以及轮廓测定等许多领域都有广泛的应用,而提取信息的精度除与测量仪器的精度有关外,还取决于所使用的分析方法,故对干涉图处理方法的研究是十分必要的。

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出,当 N_{eff}较大时,用 (17)式、(19)式得到的相对焦移较为准确,当 N_{eff}分别大于 4.8, 3.5时,相对误差小于 1%。

4 小 结

在近轴近似下直接从菲涅耳衍射积分公式出发,推导出了会聚球面波通过环形光阑衍射场分布的解析公式,对解析公式的一些特例作了讨论。此外,还用直接从轴上光强公式计算焦移和文献中已有的计算焦移的近似公式作了比较。结果表明,当 N_{eff}分别大于 4.8, 3.5时,用近似公式 (17)式、(19)式计算焦移与轴上光强公式求的焦移数值解的相对误差小于 1%。

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