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Study on green problem of SHG*

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Abstract: An extended physical model on intracavity binary wave plates based on the Jones matrix formalism that gives the prerequisite for independently intracavity frequency doubling under CW multi-longitudinal mode operation has been obtained. The formulas for eliminating the noise of the frequency doubled output in phase matching type Ⅱ and type Ⅰ conditions are presented. A general condition concerning the round-trip Jones matrix has been achieved. The eigenvectors for the phase matching type Ⅱ and type Ⅰ have been determined analytically.

Key words: sum-frequency generation(SFG) phase matching type Ⅱ and Ⅰ second harmonic generation (SHG)

关于倍频绿光问题的研究

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摘要: 采用双波片晶体的物理模型和 Jones 矩阵方法, 得到了在连续多纵模腔内倍频激光器中压缩噪声的必要条件, 推导出与Ⅱ类和Ⅰ类相位匹配方式相对应的消除和频效应的公式, 并推广为往返 Jones 矩阵元所应满足的关系, 以及相应的腔内本征矢。

关键词: 和频效应 Ⅱ类和Ⅰ类相位匹配方式 倍频效应

Introduction

Second harmonic generation (SHG) is a widely used nonlinear optical frequency conversion to generate coherent radiation in visible and UV spectral regions. However, the output noise due to large-amplitude fluctuations emerging in CW intracavity SHG lasers under multi-longitudinal

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modes operation, which has been referred to as the "green problem", has limited the development and application of compact CW solid-state SHG lasers to a great extent. In this paper, the output noise of CW SHG lasers with both phase matching type Ⅱ and type Ⅰ corresponding to the green problem for or blue problem has been investigated theoretically. As it is known, in a free-running laser of multi-longitudinal modes, such longitudinal modes oscillate simultaneously without fixed mode-to-mode amplitude and phase relationships, if there is no nonlinear crystal or mode-locking element in the cavity. The resulted laser output is a sort of time-averaged statistical mean value. Each mode oscillates independent of the others, and the phases are randomly distributed in the range of $-\pi$ to $+\pi$. In the time domain, the field consists of an intensity distribution which has the characteristics of quite small thermal noise. The total intensity is nearly the sum of these in various longitudinal modes[1]. When a frequency doubling crystal is inserted into the laser cavity, large amplitude fluctuations are observed, even if there is not any Q-switching and mode-locking elements.

Baer used a system of coupled differential equations to describe the multi-longitudinal mode laser in the rate-equation approximation and pointed out that instabilities arose from the coupling of the longitudinal modes in the laser oscillator by sum-frequency generation (SFG)[2]. The key point to stabiline the output of a CW laser with intracavity SHG is to eliminate SFG[3,4]. In this paper, a physical model based on binary wave plates (BWP) and Jones matrix has been adopted. The relationship of δ_1, δ_2 with phase shift of each wave plate and relative azimuthal angle Ψ has been determined to relieve mode coupling and eliminate the influence of SFG in nonlinear crystals for general cases of both phase matching type Ⅱ and type Ⅰ. The fundamental wave eigenvectors of round-trip Jones matrix have been presented for each phase matching type.

1 Analysis

The BWP physical model of a CW laser with intracavity SHG is shown in Fig. 1. This model shows a general case, in which the wave plate may be either frequency doubling crystal or lasing medium or another optical retarder.

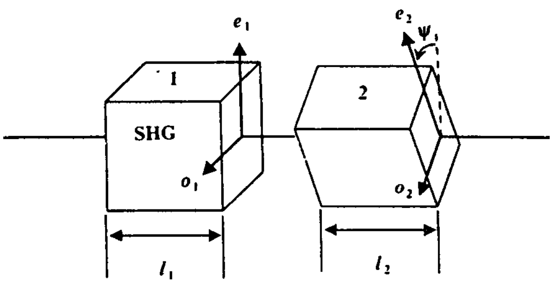

Fig. 1 The schematic diagram of BWP

A round-trip Jones matrix starting from the SHG crystal (number 1 in Fig. 1) and ending at the same point after one round trip can be expressed as

$$\boldsymbol{M} = J(0, \delta_1)J(\Psi, \delta_2)J(\Psi, \delta_2)J(0, \delta_1) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} =$$

$$\begin{bmatrix} \exp[i(\delta_1+\delta_2)]\cos^2\Psi + \exp[-i(\delta_2-\delta_1)]\sin^2\Psi & i\sin(2\Psi)\sin\delta_2 \\ i\sin(2\Psi)\sin\delta_2 & \exp[-i(\delta_1+\delta_2)]\cos^2\Psi + \exp[i(\delta_2-\delta_1)]\sin^2\Psi \end{bmatrix} \quad (1)$$

where Jones matrix of a wave plate (retarder) is

$$J(\Psi, \delta) = R(\Psi)C(\delta)R(-\Psi) = \begin{bmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{bmatrix} \begin{bmatrix} \exp(i\delta/2) & 0 \\ 0 & \exp(-i\delta/2) \end{bmatrix} \begin{bmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{bmatrix},$$

$\delta_{1,2} = 2\pi(n_{e_{1,2}} - n_{o_{1,2}}) l_{1,2}/\lambda$, λ is the fundamental laser wavelength, $n_{e_{1,2}}$ and $n_{o_{1,2}}$ are refractive indices of two major axes in each wave plate.

Two eigenvectors corresponding to $\boldsymbol{M}$ are then

$$E_1(\omega_1) = \frac{\Delta_1}{Z_1}\begin{bmatrix} b+c \\ \Delta \end{bmatrix}, \quad E_2(\omega_2) = \frac{\Delta_2}{Z_2}\begin{bmatrix} b-c \\ \Delta \end{bmatrix} \tag{2}$$

where

$$b = \cos^2\Psi\sin(\delta_1+\delta_2) - \sin^2\Psi\sin(\delta_2-\delta_1),\ c = [b^2+\Delta^2]^{1/2},\ \Delta = \sin(2\Psi)\sin\delta_2 \tag{3}$$

$$Z_1 = \sqrt{(b+c)^2+\Delta^2},\ Z_2 = \sqrt{(b-c)^2+\Delta^2} \tag{4}$$

$$\Delta_1 = A_1\exp[i(\omega_1 t+\Phi_1)],\ \Delta_2 = A_2\exp[i(\omega_2 t+\Phi_2)] \tag{5}$$

Φ_1 and Φ_2 are randomly arbitrary phase shifts. The frequencies ω_1 and ω_2 are generally very close but not identical, so that the sum frequency$(\omega_1+\omega_2)$ is very near $2\omega_1$ and $2\omega_2$[3]. Z_1 and Z_2 are normalized parameters. A_1 and A_2 are the fundamental wave field amplitudes.

For the phase matching type Ⅱ $(o_\omega + e_\omega)$, the nonlinear polarization is given by

$$P(\omega_1+\omega_2) = d_{\mathrm{eff\,II}} E_o(\omega_1,\omega_2) E_e(\omega_1,\omega_2) \tag{6}$$

where $d_{\mathrm{eff\,II}}$ is the effective nonlinear coefficient of type Ⅱ, E_o, E_e are the sum of the ordinary, extraordinary components of E_1 and E_2, respectively.

The time-average intensity with random phase approximation is[3]

$$I(\omega_1+\omega_2) = \langle P(\omega_1+\omega_2)P^*(\omega_1+\omega_2)\rangle = I_1^2\left[\frac{(b+c)\Delta}{Z_1^2}\right]^2 + I_2^2\left[\frac{(b-c)\Delta}{Z_2^2}\right] + I_1 I_2\left(\frac{2b\Delta}{Z_1 Z_2}\right)^2 \tag{7}$$

where $\langle\ \rangle$ stands for the time average.

According to dynamic analyses of rate equations[2], SHG output noise results from the SFG, which corresponds to the $I_1 I_2$ cross term in Eq. (7). Therefore, $b = 0$ is the prerequisite to eliminate the noise of output. The Eq. (3) is given in this case by

$$\cos^2\Psi\sin(\delta_1+\delta_2) - \sin^2\Psi\sin(\delta_2-\delta_1) = 0 \tag{8a}$$

or

$$\tan^2\Psi = \sin(\delta_2+\delta_1)/\sin(\delta_2-\delta_1) \qquad (0 \leqslant \Psi < \pi) \tag{8b}$$

The above equations and Fig. 2 give the relationship between δ_1, δ_2 and Ψ in the general case for the phase matching type Ⅱ to stabilize the SHG output. The eigenvectors that can satisfy $b = 0$ are $\frac{\Delta_1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\frac{\Delta_2}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, respectively and the time-average SHG output is $I(\omega_1+\omega_2) = [I^2(\omega_1) + I^2(\omega_2)]/4$ from Eqs. (2) to (8). When $\Psi = \pi/4$ and $\delta_2 = \pi/2$, $b = 0$ is always satisfied for

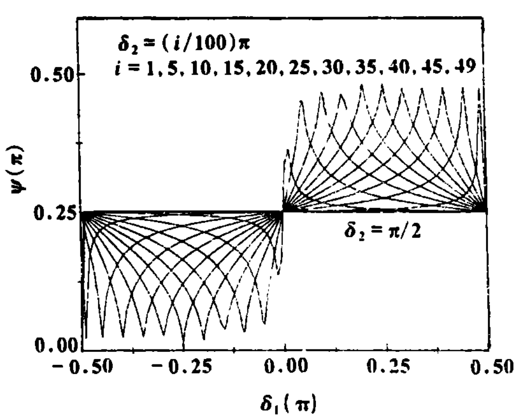

Fig. 2　Dependence of Ψ on δ_1 for different δ_2 in the case of phase matching type Ⅱ

arbitrary δ_1[4].

For the phase matching type Ⅰ, there are two cases:

(1) $o_\omega + o_\omega$ $$P(\omega_1+\omega_2)=d^{o}_{\mathrm{eff\,I}}[E_o(\omega_1,\omega_2)]^2 \tag{9}$$

$$I(\omega_1+\omega_2)=\langle P(\omega_1+\omega_2)P^*(\omega_1+\omega_2)\rangle = I_1^2\left(\frac{b+c}{Z_1}\right)^4 + I_2^2\left(\frac{b-c}{Z_2}\right)^4 + 4I_1I_2\left(\frac{\Delta^2}{Z_1Z_2}\right)^2 \tag{10}$$

(2) $e_\omega + e_\omega$ $$P(\omega_1+\omega_2)=d^{e}_{\mathrm{eff\,II}}[E_e(\omega_1,\omega_2)]^2 \tag{11}$$

$$I(\omega_1+\omega_2)=\langle P(\omega_1+\omega_2)P^*(\omega_1+\omega_2)\rangle = I_1^2\left(\frac{\Delta}{Z_1}\right)^4 + I_2^2\left(\frac{\Delta}{Z_2}\right)^4 + 4I_1I_2\left(\frac{\Delta^2}{Z_1Z_2}\right)^2 \tag{12}$$

where $d^{o}_{\mathrm{eff\,I}}$ and $d^{e}_{\mathrm{eff\,II}}$ are the effective nonlinear coefficients of type Ⅰ, respectivly.

The condition to eliminate noise for the phase matching type Ⅰ is just as for the type Ⅱ, i.e. the I_1I_2 cross term must vanish as follows: $$\Delta = \sin(2\Psi)\sin\delta_2 = 0 \tag{13}$$

The above equation shows the relationship between δ_2 and Ψ in general case for the type Ⅰ phase matching to stabilize SHG output. The eigenvectors that can satisfy $\Delta = 0$ are $\Delta_1\begin{bmatrix}1\\0\end{bmatrix}$ and $\Delta_2\begin{bmatrix}0\\1\end{bmatrix}$, and the time-average SHG output is I_1^2 or I_2^2 for the $o_\omega + o_\omega$ or $e_\omega + e_\omega$, respectively, by using Eqs. (3), (4), (10), (12) and (13).

2 Discussion

The multi-longitudinal mode laser with an intracavity SHG has been investigated by BWP model and Jones matrix. The equation system is more extensive than others[3,4] as the Ψ, δ_1 and δ_2 are varied.

For the case of phase matching type Ⅱ, equations (8) give the relationship between Ψ, δ_1 and δ_2 for eliminating SFG in general cases. When δ_2 equals $\pi/2$ using a quarter-wave plate (QWP) particularly, for instance, the azimuthal angle Ψ equals $\pi/4$ from the equations (8). This special result is the same as in Refs. [3~5]. The orthogonal eigenvectors of $\boldsymbol{M}$ is $\frac{\Delta_1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{\Delta_2}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$ yet.

As regards type Ⅰ phase matching, the relationship between Ψ and δ_2 must conform to equation (13) to eliminate SFG. It is enough when the azimuthal angel Ψ equals 0 or $\pi/2$ in accordance with Eq. (13). The eigenvectors of $\boldsymbol{M}$ should be $\Delta_1\begin{bmatrix}1\\0\end{bmatrix}$ and $\Delta_2\begin{bmatrix}0\\1\end{bmatrix}$. In the case of $o_\omega + o_\omega$ or $e_\omega + e_\omega$, the contribution to SHG output to eliminate SFG is $\Delta_1\begin{bmatrix}1\\0\end{bmatrix}$ or $\Delta_2\begin{bmatrix}0\\1\end{bmatrix}$, individually. In addition, the δ_2 can choose the $m\pi$, and this condition to eliminate SFG corresponds to inserting a full-wave plate (FWP) or a half-wave plate (HWP) into the cavity, the SHG laser will show stable output independent of Ψ. In an experimental setup, if the cavity includes only a SHG crystal and a lasing medium which can be manufactured as FWP or HWP, the SHG laser output will be stable and insensitive to the phase shift of SHG crystal and Ψ[5].

In conclusion, the conditions in formulae Eqs. (8) and (13) determine the relationship between azimuthal angle Ψ and phase shifts δ of optical elements (lasing medium, SHG crystal, wave plate, etc.) in SHG laser, to eliminate SFG influence in the cases of both type Ⅱ and type Ⅰ phase matching. In other words, if only the polarized states of fundamental wave in SHG crystal can be managed according to phase matching type Ⅱ having $\frac{\Delta_1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\frac{\Delta_2}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix}$, phase matching type Ⅰ having $\Delta_1\begin{bmatrix}1\\0\end{bmatrix}$ or $\Delta_2\begin{bmatrix}0\\1\end{bmatrix}$, respectively, for the case of $o_\omega + o_\omega$ or $e_\omega + e_\omega$, the stability of SHG output will be the same as fundamental wave's. In a more general case from the viewpoint of the round-trip Jones matrix $\boldsymbol{M}$, as long as components m_{ij} $(i, j = 1, 2)$ of $\boldsymbol{M}$ that starts from the SHG crystal and ends at the starting point after one round trip, satisfy following relationships, either $m_{11} = m_{22}$ or $m_{12} = 0 = m_{21}$, respectively for the phase matching type Ⅱ or type Ⅰ, the SFG influence can be eliminated, no matter how many numbers and kinds of optical elements are in the cavity. The relationships are in agreement with the eigenvectors of $\boldsymbol{M}$ as above.

For the case of phase matching type Ⅰ, taking a horizontal polarizer $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ as an example, $\boldsymbol{M} = \begin{bmatrix}\exp(i\delta_1)\cos^2\Psi & \sin\Psi\cos\Psi\\ \sin\Psi\cos\Psi & \exp(-i\delta_1)\sin^2\Psi\end{bmatrix}$ and $\Psi = 0$ or $\pi/2$, then $\boldsymbol{M}$ is always the diagonal matrix for the $o_\omega + o_\omega$ or $e_\omega + e_\omega$. Therefore, it is beneficial to solve "blue problem" by inserting a polarizer in an unpolarized resonator, in which the polarization of the fundamental wave radiation should be along with $\begin{bmatrix}1\\0\end{bmatrix}$ or $\begin{bmatrix}0\\1\end{bmatrix}$, according to the phase matching conditions. Above theoretical results provide foundation to solve green and blue problems.

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