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Elegant Laguerre-Gaussian beams and their properties

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Abstract: The elegant Laguerre-Gaussian (L-G) beams and their propagation properties are studied in detail using the propagation equation derived from the generalized Huygens-Fresnel diffraction integral. The beam propagation factor (M^2 factor) and power in the bucket (PIB) are used to characterize beam quality, a comparison of the beam quality of elegant and standard L-G beams is made and the results are analyzed.

Key words: propagation equation　elegant Laguerre-Gaussian (L-G) beam　M^2 factor　power in the bucket (PIB)

复宗量拉盖尔-高斯光束及其特性研究

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摘要: 用由广义惠更斯-菲涅耳衍射积分推导出的传输方程详细研究了复宗量拉盖尔-高斯光束及其传输特性。采用光束传输因子(M^2因子)和桶中功率(PIB)来表征光束质量,对复宗量和实宗量拉盖尔-高斯光束的光束质量进行了比较,并对所得结果作了分析。

关键词: 传输方程　复宗量拉盖尔-高斯光束　M^2因子　桶中功率(PIB)

Introduction

The complex-argument Hermite-Gaussian (H-G) beams, or called elegant H-G beams, were introduced by Siegman[1,2] and then recently extended to the elegant Laguerre-Gaussian (L-G) beams. The near field and far field distributions, beam propagation factor (M^2 factor) of elegant Gaussian beams, including elegant H-G and L-G beams, were studied extensively by Saghafi and Sheppard[3,4]. It was shown that elegant Gaussian modes are also solutions of the paraxial wave equation, but are not orthogonal in the usual fashion, and the arguments of the Hermite and Laguerre parts are complex. Physically, elegant Gaussian beams represent a type of beams having complex arguments, whose field profile may change form upon propagation even though in free space. The purpose of the present paper is to study elegant L-G beams and illustrate their propagation properties with numerical examples. The M^2 factor and power in the bucket (PIB) are chosen as the criteria to characterize beam quality. The beam quality of elegant and standard L-G beams is compared.

1　Propagation equations of elegant L-G beams

The field distribution of elegant L-G beams at the plane of $z = 0$ is expressed as[3]

$$E(r_0, \theta_0, z = 0) = (r_0/w_0)^m L_p^m(r_0^2/w_0^2)\exp(-r_0^2/w_0^2)\exp(-\mathrm{i}m\theta_0) \tag{1}$$

where L_p^m denotes the Laguerre polynomial with mode orders p and m, w_0 is the waist width. The propagation of elegant L-G beams through a paraxial optical $ABCD$ system is described by the generalized Huygens-Fresnel diffraction integral of the form[5]

$$E(r,\theta,z) = \frac{\mathrm{i}k}{2\pi B}\iint E(r_0,\theta_0,0)\exp\left\{-\frac{\mathrm{i}k}{2B}[Ar_0^2 - 2rr_0\cos(\theta-\theta_0) + Dr^2]\right\} r_0\mathrm{d}r_0\mathrm{d}\theta_0 \tag{2}$$

where A, B, C, D are real-valued matrix elements, which obey the relation $AD - BC = 1$.

The substitution from Eq. (1) into Eq. (2) after lengthy integral calculations yields:

$$E(r',\theta,z) = A^p\exp(-\mathrm{i}m\theta)\left(\frac{q_0/B}{1+Aq_0/B}\right)^{m+p+1} r'^m\exp\left(-\frac{q_0}{q}r'^2\right) L_p^m\left(\frac{q_0/AB}{1+Aq_0/B}r'^2\right),$$
$$(A\neq 0, A/B\neq 0) \tag{3}$$

where $r' = r/w_0$ is the normalization radial coordinate and q_0, q denote the complex q parameters at the $z=0$ and z planes, respectively, which are related by the well-known $ABCD$ law

$$q = (Aq_0 + B)/(Cq_0 + D) \tag{4}$$

Limited by the integral condition, Eq. (3) is valid for the case of $A\neq 0$ and $A/B\neq 0$. If $A = 0$ or $A/B\to 0$, Eq. (2) reduces to

$$E(r,\theta,z) = \frac{\mathrm{i}k}{2\pi B}\int_0^\infty\int_0^{2\pi} E(r_0,\theta_0,0)\exp\left\{-\frac{\mathrm{i}k}{2B}[Dr^2 - 2rr_0\cos(\theta-\theta_0)]\right\} r_0\mathrm{d}r_0\mathrm{d}\theta_0 \tag{5}$$

Substituting from Eq. (1) into Eq. (5) leads to

$$E(r',\theta) = (-1)^p(p!)^{-1}\mathrm{e}^{-\mathrm{i}m\theta}\left(\frac{q_0}{B}\right)^{2p+m+1}(r')^{2p+m}\exp\left[-\frac{q_0}{B}\left(D-\frac{q_0}{B}\right)r'^2\right],\left(A = 0 \text{ or } \frac{A}{B}\to 0\right) \tag{6}$$

Eqs. (3) and (6) are the closed-form propagation equations of elegant L-G beams, which deliver the field distribution at any plane from the Fresnel diffraction region to the far field. One of the interesting cases in practice is the focusing by a thin lens with focal length f, the corresponding $ABCD$ matrix reads

$$\begin{pmatrix} A & B\\ C & D\end{pmatrix} = \begin{pmatrix} 1 & z\\ 0 & 1\end{pmatrix}\begin{pmatrix} 1 & 0\\ -1/f & 1\end{pmatrix} = \begin{pmatrix} -\Delta z & f(1+\Delta z)\\ -1/f & 1\end{pmatrix} \tag{7}$$

with $\Delta z = (z-f)/f$, Eqs. (3) and (6) reduce to

$$E_2(r',\theta,\Delta z) = \mathrm{e}^{-\mathrm{i}m\theta}(-\Delta z)^p\left[\frac{\mathrm{i}\pi N_f}{1+\Delta z(1-\mathrm{i}\pi N_f)}\right]^{m+p+1} r'^m L_p^m\left[\frac{-\mathrm{i}\pi N_f r'^2}{\Delta z[1+\Delta z(1-\mathrm{i}\pi N_f)]}\right]\times$$
$$\exp\left[-\frac{\mathrm{i}\pi N_f(1-\mathrm{i}\pi N_f)}{1+\Delta z(1-\mathrm{i}\pi N_f)}r'^2\right],\quad (z\neq f) \tag{8}$$

and

$$E_{pm}(r',\theta,\Delta z = 0) = (-1)^p(p!)^{-1}\exp(-\mathrm{i}m\theta)(\mathrm{i}\pi N_f)^{2p+m+1}r'^{2p+m}\times$$
$$\exp\left[-\mathrm{i}\pi N_f(1-\mathrm{i}\pi N_f)r'^2\right],\quad (z = f) \tag{9}$$

respectively, where

$$N_f = w_0^2/\lambda f \tag{10}$$

is the Fresnel number.

Numerical calculations were performed by means of Eqs. (8) and (9), typical examples are given in Figs. 1 and 2, where the calculation parameters are $N_f = 5$, (a) $p = 0$, (b) $p = 1$, (c) $p = 2$, (d) $p = 3$. The normalization factor $I_0 = \pi w_0^2/2$ is the integrated irradiance of the fundamental Gaussian beam at the $z = 0$ plane. For the convenience of comparison, the results of the corresponding standard L-G beams are complied in Figs. 3 and 4 using the well-known

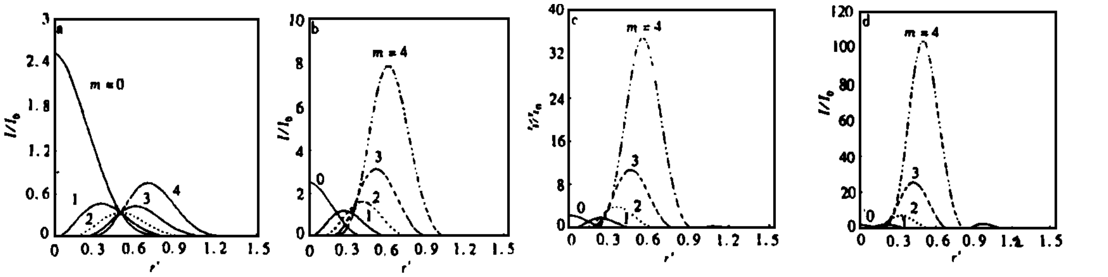

Fig. 1 Relative irradiance distributions at the plane of $\Delta z = -0.5$ of elegant L-G beams focused by a lens, the calculation parameters are $N_f = 5$ a- $p = 0$ b- $p = 1$ c- $p = 2$ d- $p = 3$

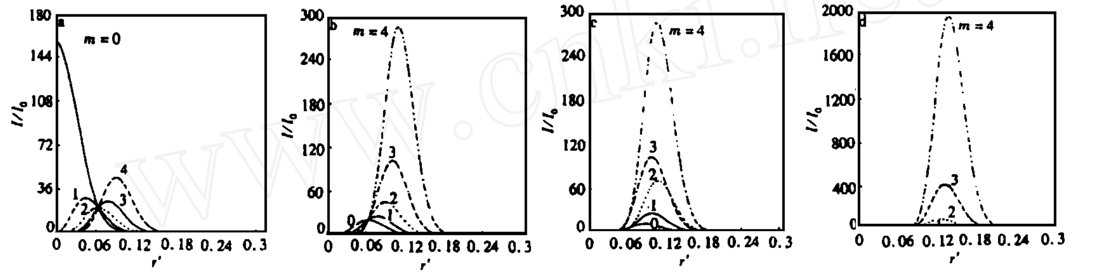

Fig. 2 Relative irradiance distributions at the focal plane of $\Delta z = 0$ of elegant L-G beams focused by a lens, the calculation parameters are $N_f = 5$ a- $p = 0$ b- $p = 1$ c- $p = 2$ d- $p = 3$

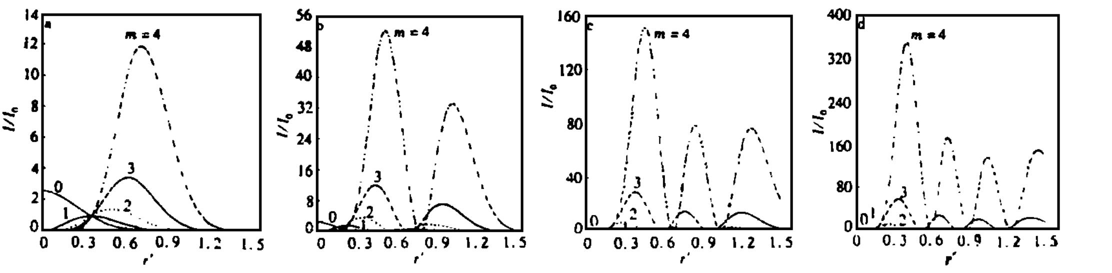

Fig. 3 Relative irradiance distributions at the plane of $\Delta z = -0.5$ of standard L-G beams focused by a lens, the calculation parameters are $N_f = 5$ a- $p = 0$ b- $p = 1$ c- $p = 2$ d- $p = 3$

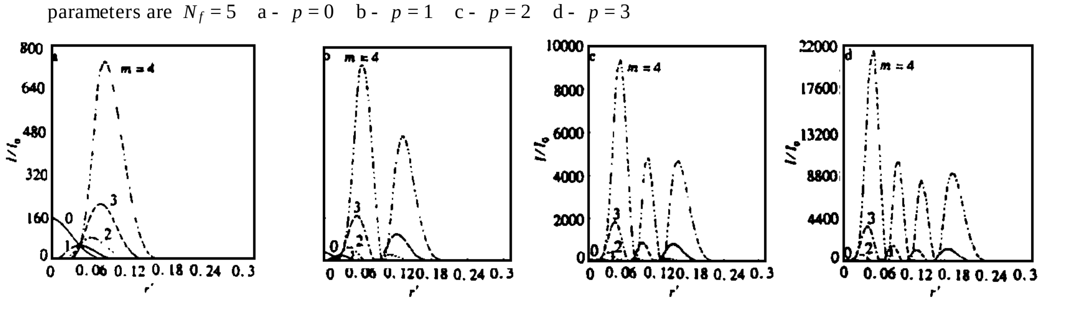

Fig. 4 Relative irradiance distributions at the focal plane of $\Delta z = 0$ of standard L-G beams focused by a lens, the calculation parameters are $N_f = 5$ a- $p = 0$ b- $p = 1$ c- $p = 2$ d- $p = 3$

propagation formula (for example, see Eq. (13) in Ref. [6]). It turns out readily from Figs. 1～4 that, except for the modes of $p = 0$, $m = 0, 1 \cdots$, which are indistinguishable from the standard L-G modes, the propagation and focusing behaviors of elegant L-G beams are notably different from those of standard L-G beams. The standard L-G beams retain their irradiance profile unchanged upon propagation, whereas the irradiance profiles of elegant L-G beams vary not only at the focal plane of $\Delta z = 0$, but also at the plane of $\Delta z = -0.5$. Moreover, it is shown that our approach provides possibility of simulating the propagation of elegant L-G beams from one plane to another as they propagate through a paraxial $ABCD$ system.

2 M^2 factor and PIB

For some practical applications, the M^2 factor and/or PIB are often used to characterize laser beam quality[7,8].

2.1 M^2 factor

The M^2 factor of standard L-G beams reads[8]

$$M^2 = 2p + m + 1 \tag{11}$$

However, it is difficult to derive an analytical expression for the M^2 factor of elegant L-G beams, but numerical calculations based on the second-moments definition[7] are still available.

2.2 PIB

The PIB is defined as the fractional power within a given bucket's size in the far field, and is written as

$$\mathrm{PIB} = \int_0^{2\pi}\int_0^{a/w_0} |E(r', \theta, z = f)|^2 r' \mathrm{d}r' \mathrm{d}\theta \Big/ \int_0^{2\pi}\int_0^{\infty} |E(r', \theta, z = f)|^2 r' \mathrm{d}r' \mathrm{d}\theta \tag{12}$$

where a is the bucket's width, and $E(r', \theta, f)$ is given in Eq. (8). Numerical comparative results are plotted in Figs. 5 and 6, from which it follows that, apart from $p = 0$, $m = 0, 1$, the M^2 factor of elegant L-G beams is less than that of standard L-G beams, and increases nearly equally with m, but increases more slowly with p. Thus, the beam quality of elegant L-G beams measured by the M^2 factor is better than that of standard L-G beams. However, as shown in Fig. 6, this is not always true, providing that the PIB is chosen as the criterion to characterize beam quality. The PIB of the elegant L-G beam of order $p = 2$, $m = 1$ is less than that of the standard L-G beam within the region $0 < r' < 0.108$, on the contrary, the elegant L-G beam shows the more PIB than the standard one within $0.108 < r' < 0.2$. This can be understood well by virtue of Fig. 7, where irradiance distributions of the standard and

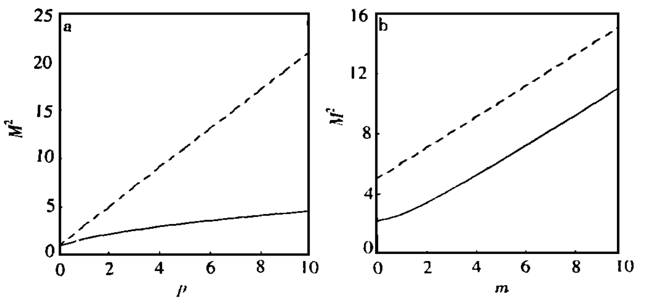

Fig. 5 M^2 factor of elegant (——) and standard (——) L-G beams as a function of order a - p ($m = 0$) b - m ($p = 2$)

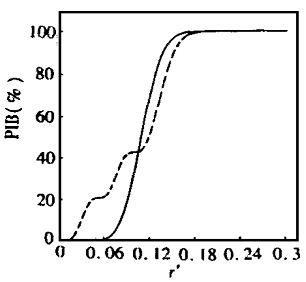

Fig. 6 PIB curves versus bucket's size r' for elegant (——) and standard (——) L-G beams, $N_f = 5$, $p = 2$, $m = 1$

elegant L-G beams of orders $p = 2$, $m = 1$ are complied together. The physical reason is that the PIB is dependent upon the irradiance distribution in the far field and the chosen bucket's size.

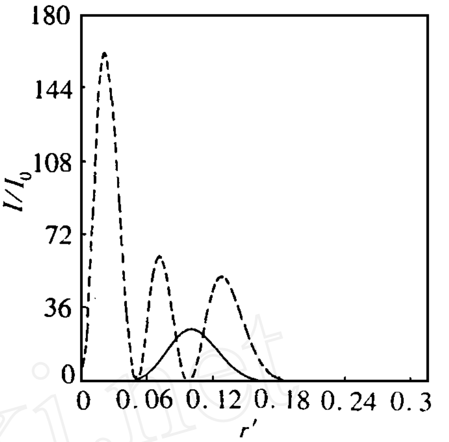

Fig. 7 Relative irradiance distributions at the focal plane ($\Delta z = 0$) of elegant (——) and standard (——) L-G beams, $N_f = 5$, $p = 2$, $m = 1$

3 Conclusion remarks

The propagation properties of elegant L-G beams, including beam quality and irradiance distributions at any propagation plane from the Fresnel diffraction region to the far field, have been studied and analyzed in detail. Numerical calculation results have demonstrated the different propagation properties of elegant and standard L-G beams. A comparative study of the beam quality of elegant and standard L-G beams has shown that different conclusion may be drawn, depending upon the criterion and the chosen bucket's size. The results can be extended to the elegant H-G beams, which is found in Ref. [9] for the one-dimensional case. Furthermore the extension to the two-dimensional case is straightforward. However, for the apertured case, it is difficult to find analytical results for elegant Gaussian beams, but Eqs. (1) and (2) are still applicable to performing numerical calculations.

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