1. Definition of fundamental mode thermally stable resonators

The definition of fundamental mode thermally stable resonators was given in the literature [1]. With its necessary conditions being:

$$d\alpha/df = 0 \tag{1}$$

$$dV/df = 0 \tag{2}$$

where α is the beam divergence angle, V is the mode volume and f is the thermal focal length. On the premise that the beam wavefront is coincident with the curved surface of the resonator mirror, (1) and (2) can be equivalent to

$$dw/df = 0 \tag{3}$$

where w is the spot radius on the output mirror. All discussions made in are based on (3), but (3) as the definition of fundamental mode thermally stable resonators is conditional, i.e. coincidence of the beam wavefront with the mirror surface. For ring resonators, resonators with Gaussian reflectivity mirrors of conventional resonators with variable reflectivity mirrors, the beam wavefront is not always coincident with the mirror surface and varies with the thermal focal length, hence the problem of thermal stability of modes can not be simplified into that of spot radius w to be considered. Therefore, the result from [1] is only suited to simple stationary wave resonators. Based on detailed considerations of thermal stability of the fundamental mode, we define

$$d(1/q)/df = 0 \tag{4}$$

as the stability condition of fundamental mode thermally stable resonators, where q is the complex parameter of a Gaussian beam. In accordance with the definition, equation (4) can be resolved into

$$dw/df = 0 \tag{5}$$

$$dR/df = 0 \tag{6}$$

where R is the curvature radius of the beam wavefront at the output mirror surface. It is obvious that, when the beam wavefront is coincident with the mirror surface, R is constant, equation (6) automatically holds. Therefore, fundamental mode thermally stable resonators defined by us include the situation, i.e. the problem of thermal stability of simple stationary wave resonators, discussed in the literature [1].

In addition, our definition can be proved to be equivalent to that in [1]. In accordance with [1].

$$V = \pi d w^2\left[\left(1-\frac{a}{\rho_1}\right)^2+\left(\alpha\,\frac{\lambda}{\pi w^2}\right)^2\right] \tag{7}$$

where definitions of all parameters are referred to [1]. Let us differentiate (7) with respect to f

$$dV/df = f_1(w)\cdot dw/df \tag{8}$$

Obviously, when (5) holds, (2) also does. Suggest the complex parameter of a Gaussian beam at the output mirror surface be

$$1/q = 1/R - i\lambda/(\pi w^2) \tag{9}$$

In accordance with the law $ABCD$, after q pass through $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, its complex parameter is

$$\frac{1}{q_1}=\frac{\left(1+\frac{l}{R}\right)\frac{1}{R}+\left(\frac{\lambda}{\pi w w_1}\right)^2 l}{\left(1+\frac{l}{R}\right)^2+\left(\frac{\lambda l}{\pi w^2}\right)^2}-i\frac{\lambda}{\pi}\cdot\frac{1}{w^2\left[\left(1+\frac{l}{R}\right)^2+\left(\frac{\lambda l}{\pi w^2}\right)^2\right]} \tag{10}$$

In accordance with the definition of a divergence angle in the far field:

$$a=\lim_{l\to\infty}(w_1/l)=w\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{\lambda}{\pi w}\right)^2} \tag{11}$$

differentiation of (11) with respect to f leads to

$$\mathrm{d}a/\mathrm{d}f=f_2(w,R)\cdot\mathrm{d}w/\mathrm{d}f+f_3(w,R)\cdot\mathrm{d}R/\mathrm{d}f \tag{12}$$

It is known from (12) that, when (5) holds and if (6) also does, (1) does. Therefore, the two definitions are equivalent to each other.

In general, resonators can be divided into two groups, i.e. stable and unstable resonators. The problem of thermal stability of unstable resonators had been discussed by us in literatures [2] and [3] in detail. In literatures [4] and [5], we had proved that resonators with Gaussian reflectivity mirrors (GRM) could not realize unstable resonators, i.e. all eigen-modes in GRM resonators are Gaussian beams. Therefore, the situation in which eigen modes in resonators are Gaussian beam is only analyzed in the paper.

2. Modes in GRM resonators

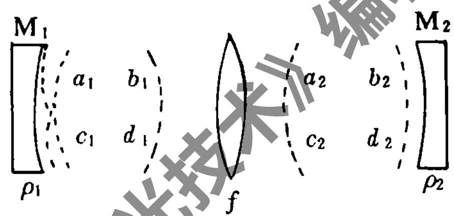

Fig. 1 GRM resonator with thermal lens

As shown in Fig. 1, the active medium is made equivalent to a thin lens, and optics on its both sides are described by

$$\begin{bmatrix}a_1 & b_1\\ c_1 & d_1\end{bmatrix},\quad \begin{bmatrix}a_2 & b_2\\ c_2 & d_2\end{bmatrix} \tag{13}$$

respectively. The single-pass propagation matrix in the resonators is

$$\begin{aligned}\begin{bmatrix}a & b\\ c & d\end{bmatrix}&=\begin{bmatrix}a_2 & b_2\\ c_2 & d_2\end{bmatrix}\begin{bmatrix}1 & 0\\ -1/f & 1\end{bmatrix}\begin{bmatrix}a_1 & b_1\\ c_1 & d_1\end{bmatrix}\\ &=\begin{bmatrix}a_1a_2+b_2c_1-a_1b_2/f & a_2b_1+b_2d_1-b_1b_2/f\\ a_1c_2+c_1d_2-a_1d_2/f & b_1c_2+d_1d_2-b_1d_2/f\end{bmatrix}\end{aligned} \tag{14}$$

With the mirror M_1 as reference, the round-trip propagation matrix is

$$\begin{aligned}\begin{bmatrix}A & B\\ C & D\end{bmatrix}&=\begin{bmatrix}1 & 0\\ -2/\rho_1-i\lambda/(\pi\sigma^2) & 1\end{bmatrix}\begin{bmatrix}d & b\\ c & a\end{bmatrix}\begin{bmatrix}1 & 0\\ -2/\rho_2 & 1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}\\ &=\begin{bmatrix}1 & 0\\ -2/\rho_1-i\lambda/(\pi\sigma^2) & 1\end{bmatrix}\begin{bmatrix}A_r & B_r\\ C_r & D_r\end{bmatrix}\\ &=\begin{bmatrix}A_r & B_r\\ C_r-2A_r/\rho_1-i\lambda A_r/(\pi\sigma^2) & D_r-2B_r/\rho_1-i\lambda B_r/(\pi\sigma^2)\end{bmatrix}\end{aligned} \tag{15}$$

where

$$\begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} = \begin{bmatrix} ad + bc - \dfrac{2ab}{\rho_2} & 2bd - \dfrac{2b^2}{\rho_2} \\ 2ac - \dfrac{2a^2}{\rho_2} & ad + bc - \dfrac{2ab}{\rho_2} \end{bmatrix} \tag{16}$$

obviously,

$$A_r = D_r$$
$$A_r D_r - B_r C_r = 1 \tag{17}$$

In accordance with the self-consistent condition,

$$1/q = (C + D/q)/(A + B/q) \tag{18}$$

we order

$$1/q = 1/R - i\lambda/(\pi w^2) \tag{19}$$

and substitute (15) and (19) into (18) and separate real part of (18), obtaining the imaginary part

$$2\sigma^2(1/R + 1/\rho_1) = w^2(1/R + A_r/B_r) \tag{20}$$

$$(1/R + 1/\rho_1)^2 - (1/\rho_1 - A_r/B_r)^2 + (\lambda/\pi w)^2(1/\sigma^2 - 1/w^2) + 1/B_r^2 = 0 \tag{21}$$

The spot radius w of a Gaussian beam and the curvature radius R of a wave front can be derived from (20) and (21).

3. Thermal stability conditions of GRM resonators

In accordance with (5) and (6), derivation of (20) and (21) with respect to f results in

$$\mathrm{d}(B_r/A_r)/\mathrm{d}f = 0 \tag{22}$$

$$2(1/\rho_1 - A_r/B_r)\mathrm{d}(A_r/B_r)/\mathrm{d}f + \mathrm{d}(1/B_r^2)/\mathrm{d}f = 0 \tag{23}$$

Arrangement of (22) and (23) has

$$\mathrm{d}A_r/\mathrm{d}f = 0 \tag{24}$$

$$\mathrm{d}B_r/\mathrm{d}f = 0 \tag{25}$$

In accordance with (14) and (16), solving (24) and (25) can have

$$G_2\mathrm{d}a/\mathrm{d}f + a\mathrm{d}G_2/\mathrm{d}f = 0 \tag{26}$$

$$G_2\mathrm{d}b/\mathrm{d}f + b\mathrm{d}G_2/\mathrm{d}f = 0 \tag{27}$$

(27) × a − (26) × b has (where $G_2 = d - b/\rho_2$)

$$(a\mathrm{d}b/\mathrm{d}f - b\mathrm{d}a/\mathrm{d}f)G_2 = 0 \tag{28}$$

that is, thermal stability conditions of GRM resonators are

$$G_2 = 0 \tag{29}$$

or

$$a\mathrm{d}b/\mathrm{d}f - b\mathrm{d}a/\mathrm{d}f = 0 \tag{30}$$

Substitution of (14) into (30) will have

$$(a_1 d_1 - b_1 c_1)b_2^2 = 0 \tag{31}$$

Beacause

$$a_1 d_1 - b_1 c_1 = 1 \tag{32}$$

$$b_2^2 = 0 \tag{33}$$