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Matrix methods for analysing optical resonators (Part 3)

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分析光学谐振腔的矩阵方法(3)

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Matrix formulation of unstable resonators

The matrix formulation in Sec.2 can be generalized to analysing unstable resonators only replacing the q parameter by the curvature radius of spherical eigenwave as the resonator characteristic parameter. The self-consistency condition demands

$$r_1=\frac{Ar_1+B}{Cr_1+D} \tag{71}$$

where r_1 is the curvature radius of eigenwave on the mirror M_1 (Fig.1) and A, B, C, D are round-trip matrix elements.

Similar way as in Sec.2 leads to the curvature radii r_1, r_2 of eigenwave on the mirrors M_1, M_2, i.e. the positions of conjugate imaging points referred to the mirrors M_1, M_2, respectively, as follows

$$\frac{1}{r_1}=\frac{D-A}{2B}\pm\frac{1}{B}\sqrt{\left(\frac{A+D}{2}\right)^2-1}$$

$$=-\frac{1}{R_1}\pm\frac{1}{bG_2}\sqrt{G_1G_2(G_1G_2-1)} \tag{72a}$$

or

$$r_1=\frac{G_2(a-G_1)\pm\sqrt{G_1G_2(G_1G_2-1)}}{2aG_1G_2-a^2G_2-G_1}b \tag{72b}$$

$$\frac{1}{r_2}=\frac{D'-A'}{2B'}\pm\frac{1}{B'}\sqrt{\left(\frac{A'+D'}{2}\right)^2-1}$$

$$= -\frac{1}{R_2} \pm \frac{1}{bG_1}\sqrt{G_1G_2(G_1G_2-1)} \tag{73a}$$

or

$$r_2 = \frac{G_1(d-G_2) \pm \sqrt{G_1G_2(G_1G_2-1)}}{2dG_1G_2 - d^2G_1 - G_2}\, b \tag{73b}$$

where the symbol meaning is the same as in Sec. 2.

The magnification per round trip M is

$$M = \frac{A+D}{2} \pm \sqrt{\left(\frac{A+D}{2}\right)^2 - 1}$$

$$= 2G_1G_2 - 1 \pm 2\sqrt{G_1G_2(G_1G_2-1)} \tag{74}$$

and the diffraction coupling $\Gamma = 1 - 1/M^2$ (75)

The unstable condition is expressed in terms of the $ABCD$ matrix or G parameters as

$(A+D)/2 > 1$, i.e. $G_1G_2 > 1$ (for positive branch)

or (76)

$(A+D)/2 < -1$, i.e. $G_1G_2 < 0$ (for negative branch)

Eqs. (72) ~ (76) and the $ABCD$ law give a complete matrix formulation for unstable resonators. The positive sign before the radical in (72) ~ (75) is selected for positive branch, and the negative sign for negative branch.

Confocal multielement unstable resonator (CMUR)

In unstable-resonator design, one is often interested in the confocal type to obtain a near-diffraction limited output beam, therefore a confocal crossed-prism resonator is taken as an illustrated example, where the collimated beam is coupled from the polarizing plate inside the resonator, whose equivalent configuration is shown in Fig. 6a. The resonator matrix **per round trip referred to the lens f_1** is

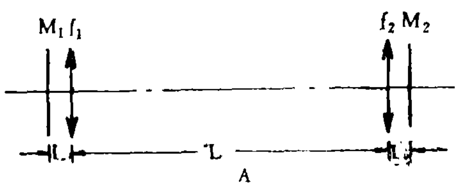

Fig.6a An equivalent resonator of the confocal crossed prism resonator

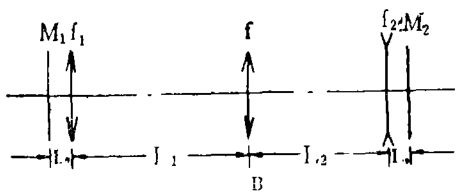

Fig.6b An equivalent resonator of the confocal crossed prism resonator with a thermal lens

$$M=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ -1/f_1 & 1\end{pmatrix}\begin{pmatrix}1 & 2l_0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -1/f_1 & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -1/f_2 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 2l_0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ -1/f_2 & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix} \qquad (77)$$

Substituting (77) into the generalized confocal condition[4]

$$AD=1 \qquad (78)$$

we get

$$\delta=\frac{1}{2}\left(\frac{f_1^2}{l_0-f_1}+\frac{f_2^2}{l_0-f_2}\right) \qquad (79)$$

and

$$\delta=L-f_1-f_2 \qquad (80)$$

The equations which determine positions of conjugate points are

$$r_1=f_1-l_0,\quad r_2=l_0-f_2 \qquad (81)$$

The magnification M, diffraction coupling Γ, and unstable condition are formally the same as (74) (75) and (76), where the G parameters are expressed by

$$G_1=\frac{(l_0-f_2)\delta-f_2^2}{f_1f_2} \qquad G_2=\frac{(l_0-f_1)\delta-f_1^2}{f_1f_2} \qquad (82)$$

Under high-power pumping the thermal lensing has to be considered, and the equivalent resonator now is one in Fig. 6b, where f is the focal length of thermal lens, thus the design equation of CMUR is expressed in terms of resonator parameters as

$$f^2(l_0-f_1)[2(l_0-f_2)\delta_2-f_2^2]$$
$$=\delta_2[(l_0-f_2)\delta_2-f_2^2][2(l_0-f_1)\delta_1-f_1^2] \qquad (83)$$

where
$$\delta_1=l_1-f-f_1$$
$$\delta_2=l_2-f-f_2 \qquad (84)$$

and the positions of conjugate points

$$r_1=f_1-l_0$$
$$r_2=l_0-f_2-f_2^2/\delta_2 \qquad (85)$$

the resonator G parameters read

$$G_1=\frac{(f^2-\delta_1\delta_2)(l_0-f_2)+f_2^2\delta_1}{f_1f_2f}$$

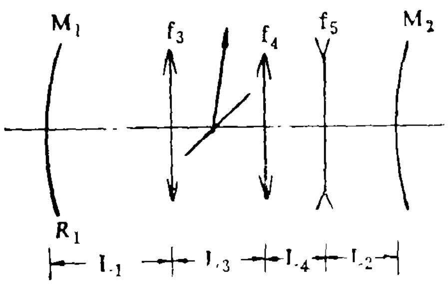

Fig.7 A generalized confocal multielement unstable resonator

$$G_2=\frac{(f^2-\delta_1\delta_2)(l_0-f_1)+f_1{}^2\delta_2}{f_1f_2f} \tag{86}$$

For a more complicated resonator configuration shown in Fig.7, tedious matrix calculations are substituted, preferably, by iterating Newton imaging equation[22], the generalized confocal design equation corresponding to (78) is

$$\begin{aligned}&(\delta_1{}^2\delta_3-\delta_1f_3{}^2-\delta_3f_1{}^2)\left[(\delta_2\delta_4-f_5{}^2)^2-f_2{}^2\delta_4{}^2\right]\\&=f_4{}^2(\delta_1{}^2-f_1{}^2)\left[(\delta_2\delta_4-f_5{}^2)\delta_2-f_2{}^2\delta_4\right]\end{aligned} \tag{87}$$

where

$$\begin{cases}\delta_1=l_1-(f_1+f_3)\\ \delta_2=l_2-(f_2+f_5)\\ \delta_3=l_3-(f_3+f_4)\\ \delta_4=l_4-(f_4+f_5)\end{cases} \tag{88}$$

and

$$f_1=R_1/2\ , \qquad f_2=R_2/2 \tag{89}$$

The positions of conjugate points read

$$r_1=-(f_1+\delta_1) \tag{90}$$

$$r_2=-f_2-\frac{f_2{}^2\delta_4}{\delta_2\delta_4-f_5{}^2}$$

The resonator G parameters are written

$$\begin{aligned}G_1&=a-b/2f_1\\ G_2&=d-b/2f_2\end{aligned} \tag{91}$$

where

$$a=c(\delta_2+f_2)+\frac{f_5\delta_3}{f_3f_4}$$

$$b=a(\delta_1+f_1)+\frac{f_3\left[(\delta_2+f_2)\delta_4-f_5{}^2\right]}{f_4f_5} \tag{92}$$

$$c=\frac{f_4{}^2-\delta_3\delta_4}{f_3f_4f_5}$$

$$d=c(\delta_1+f_1)+\frac{f_3\delta_4}{f_4f_5}$$

The expressions (87) ~ (92) are suitable for computerized resonator design.

Conclusion

We have shown that matrix methods are very useful to describe beam transformation in optical resonators. The interesting results obtained in treating TRTL, CPR and CMUR could find some applications to resonator engineering.

The axially asymmetric resonator described by a 4×4 expanded matrix [23] are not considered in this paper. Moreover, computerized numerical calculations and experimental verifications of the theory are also not compiled here. Nevertheless, some results can be found in [6,8,14].

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