Matrix methods for analysing optical resonators (Part 2)

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分析光学谐振腔的矩阵方法(2)

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Telescopic resonator with a thermal lens (TRTL)

A typical TRTL is formed by two mirrors with radius of curvature R_1, R_2 and a telescope composed by two lenses of focal length f_1, f_2 separated by l and an internal thermal lens with variable focal length f. The other parameters are denoted in Fig.3. To simplify analysis, it will be useful to replace the lens combination(f, f_1, f_2) by a thick lens whose distances of the principal planes h_1, h_2 and focal length F are

$$h_1=(lf_1-\delta l')f/[\delta(l'-f-f_1)-f_1^2]$$

$$h_2=[l(f_1+f)-l'(f_2+\delta)]f_2/[\delta(l'-f-f_1)-f_1^2] \quad (25)$$

$$F=f_1f_2f/[\delta(l'-f-f_1)-f_1^2] \quad (26)$$

where $\delta=l-f_1-f_2$ (27)

is the telescope defocusing.

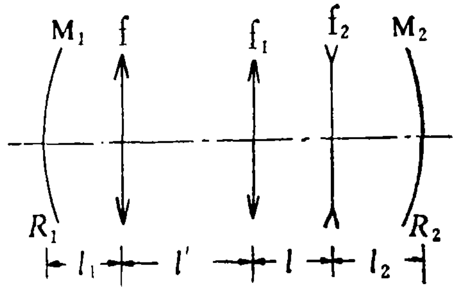

Fig.3 A telescopic resonator with a thermal lens

By removing reference planes, i.e. $l_1\to\tilde{l}_1=l_1+h_1$, $l_2\to\tilde{l}_2=l_2+h_2$ the new equivalent resonator of the TRTL in Fig.3 contains only one thin lens of focal length F, this is a well known resonator studied by Weber et al.[3], therefore the beam parameters are formally the same as (5) (6) (8) (9) (10)~(13), the stability condition (15) and misalignment sensitivity (24) are retained unchanged, only the effective resonator length b and G parameters in the formulae should be replaced by

$$b = \tilde{l}_1 + \tilde{l}_2 - \tilde{l}_1 \tilde{l}_2 / F \tag{28}$$

$$G_i = 1 - \frac{\tilde{l}_1 + \tilde{l}_2}{R_i} - \frac{\tilde{l}_j}{F}\left(1 - \frac{\tilde{l}_i}{R_i}\right) \tag{29}$$

which are variable with the thermal focal length f.

The condition for dynamic stability (20) holds true, but

$$\begin{aligned} b_1 &= l_1 \\ b_2 &= \bar{l}_2 + \bar{l} - \bar{l}_2 \bar{l} / f' \\ f' &= -f_1 f_2 / \delta \\ \bar{l} &= l' - l f_1 / \delta \\ \bar{l}_2 &= l_2 - f_2 l / \delta \end{aligned} \tag{30}$$

A special case of (20) is

$$b_1 = l_1 = 0, \quad G_1 G_2 = 1/2 \tag{31}$$

This is a well-known formula used for design of dynamic stable resonators[10,12].

In practical resonator design, it is often taken an interest in search for an analytical expression of the telescope defocusing δ which retains the condition (31) unchanged. Taking into account (27) (29) (31), we have shown that δ is a root of the equation[17,18]

$$\alpha\delta^2 + \beta\delta - \gamma = 0 \tag{32}$$

i.e.

$$\delta = \left(-\beta \pm \sqrt{\beta^2 + 4\alpha\gamma}\right) / 2\alpha \tag{33}$$

where

$$\alpha = \left[\frac{(l_2 - f_2)(f_1 + f - l')}{f_1 f_2 f} + \frac{1}{R_1}\left(1 - \frac{l_2}{f_2}\right)\left(\frac{l'}{f_1} - 1\right)\right]\left[\frac{l' - f_1}{f_1 f_2} + \frac{1}{R_2}\left(1 - \frac{l_2}{f_2}\right)\left(\frac{l'}{f_1} - 1\right)\right]$$

$$\beta = \left[\frac{f_1^2(l_2 - f_2) - f_2^2(f + f_1 - l')}{f_1 f_2 f} - \frac{1}{R_1}\left(f_1 + f_2 - \frac{f_1 l_2}{f_2} - \frac{f_2 l'}{f_1}\right)\right]\left[\frac{l' - f_1}{f_1 f_2}\right.$$

$$+\frac{1}{R_2}\left(1-\frac{l_2}{f_2}\right)\left(\frac{l'}{f_1}-1\right)\Big]-\Big[\frac{f_1}{f_2}+\frac{1}{R_2}\left(f_1+f_2-\frac{f_1 l_2}{f_2}-\frac{f_2 l'}{f_1}\right)\Big]$$

$$\Big[\frac{(l_2-f_2)(f_1+f-l')}{f_1 f_2 f}+\frac{1}{R_1}\left(1-\frac{l_2}{f_2}\right)\left(\frac{l'}{f_1}-1\right)\Big]$$

$$\gamma=\Big[\frac{f_1^2(l_2-f_2)-f_2^2(f_1+f-l')}{f_1 f_2 f}-\frac{1}{R_1}\left(f_1+f_2-\frac{f_1 l_2}{f_2}-\frac{f_2 l'}{f_1}\right)\Big]$$

$$\Big[\frac{f_1}{f_2}+\frac{1}{R_2}\left(f_1+f_2-\frac{f_1 l_2}{f_2}-\frac{f_2 l'}{f_1}\right)\Big]+\frac{1}{2} \tag{34}$$

Under the condition $R_1\to\infty$, $R_2\to\infty$, (34) becomes

$$\alpha=\frac{(l'-f_1)(l_2-f_2)(f_1+f-l')}{f_1^2 f_2^2 f}$$

$$\beta=\frac{(l'-f_1)[f_1^2(l_2-f_2)-f_2^2(f_1+f-l')]-f_1^2(l_2-f_2)(f_1+f-l')}{f_1^2 f_2^2 f}$$

$$\gamma=\frac{f_1^2(l_2-f_2)-f_2^2(f_1+f-l')+f_2^2 f/2}{f_2^2 f} \tag{35}$$

If we adopt the approximation $l'-f_1\cong 0$ used by Hanna[9], Eq.(33) reduces to

$$\delta\cong f_1^2\Big[\frac{f_2^2}{2f_1^2(l_2-f_2)}\cdot\frac{1-2(l_2-f_2)/R_2}{1-(l_2-f_2)/R_2}-\frac{1}{f}-\frac{1}{R_1}\Big] \tag{36}$$

Furthermore, if $R_1\to\infty$, $R_2\to\infty$, then

$$\delta\cong f_1^2\Big[\frac{f_2^2}{2f_1^2(l_2-f_2)}-\frac{1}{f}\Big] \tag{37}$$

(36), (37) are just the same as (25), (28) in[10] derived by Hanna et al. In comparison with (33), (36), (37), Eq.(33) is an exact formula for the telescope defocusing which can compensate the thermal lensing of laser rod under the condition(31).

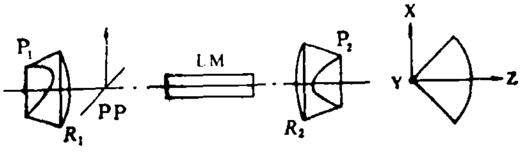

Fig.4a A crossed-prism resonator
P_1, P_2—prism LM—laser medium
PP—polarizing plate

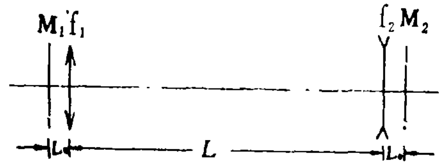

Fig.4b An equivalent resonator of Fig.4a

Crossed-prism resonator (CPR)

A CPR plotted in Fig.4a consists of two prisms P_1, P_2 with curvature radius R_1, R_2, respectively, separated by a distance L, prism thickness is d, and laser output is coupled from a polarizing plate. According to the resonator matrix theory, this configuration can be equivalent to a plane-plane multielement resonator shown in Fig.4b, where the focal length of two internal lenses is f_1, f_2, respectively

$$f_i = R_i / (n-1) \tag{38}$$

and $l_0 = d/n$, n is refractive index of prism

The waist radius W_{00} between the lenses f_1 and f_2, which determines the beam divergence angle

$$\theta_{00} = \lambda / \pi W_{00} \tag{40}$$

can be derived by using the "hybrid" equivalent resonator characterized by g'^* parameters[9,10]

$$W_{00}^2 = \frac{\lambda L'^*}{\pi} \frac{[g_1'^* g_2'^*(1-g_1'^* g_2'^*)]^{1/2}}{g_1'^*(L'^*/P_2')^2 + g_2'^*(1-g_1'^* g_2'^*)} \tag{41}$$

and its position L_{01} referred to M_1

$$L_{01} = l_0 + l_2' - L'^* \frac{g_1'^* L'^* / p_2'}{g_1'^*(L'^*/p_2')^2 + g_2'^*(1-g_1'^* g_2'^*)} \tag{42}$$

where

$$L'^* = l_0 + l_2' - l_0 l_2' / f_1 \tag{43}$$

$$g_1'^* = 1 - l_2' / f_1$$
$$g_2'^* = 1 - \frac{l_0 + l_2'}{p_2'} - \frac{l_0}{f_1}\left(1 - \frac{l_2'}{p_2'}\right) \tag{44}$$

$$p_2' = f_2^2 / (f_2 - l_0) \tag{45}$$

$$l_2' = L - f_2 + f_2^2 / (f_2 - l_0) \tag{46}$$

The stability condition expressed in terms of g'^* parameters as

$$0 < g_1'^* g_2'^* < 1 \tag{47}$$

Noting that the inequality

$$l_0 \ll L,\ R_1,\ R_2 \tag{48}$$

is always fulfilled in practical laser systems, prism transfer matrixes in the xoz and yoz planes (Fig.4a) can be written approximately as

$$T_{xoz,i} = \begin{pmatrix} -1 & 0 \\ 2/R_i' & -1 \end{pmatrix} \tag{49}$$

$$T_{yoz,i} = \begin{pmatrix} 1 & 0 \\ -2/R_i' & 1 \end{pmatrix} \tag{50}$$

where $R_i' = R_i/(n-1) = f_i \quad (i=1,\ 2)$ (51)

and the augmented matrixes corresponding to (49), (50) are

$$T_{xoz,i} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 2/R_i' & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{52}$$

$$T_{yoz,i} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2/R_i' & 1 & 0 & -2\alpha_{iy} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{53}$$

where α_{iy} is the rotating angle of the prism P_i around the x axis.

From (52), (53) and the definition of D_i, straightforward matrix calculations deliver

$$D_{xoz,i} = 0$$

$$D_{yoz,i}^2 = \frac{\pi L}{\lambda}\left(\frac{g_j'}{g_i'}\right)^{1/2} \frac{1+g_1'g_2'}{(1-g_1'g_2')^{3/2}} \quad (i,\ j=1,2 \quad i \neq j) \tag{54}$$

where the g' parameters are

$$g_i' = 1 - L/R_i' \tag{55}$$

Eq.(54) means that the misalignment characteristics around the x axis of a stable prism resonator is similar to those of spherical resonator, which is more insensitive in comparison with plane-plane one. In addition, under the condition of small misalignment the prism resonator around the y axis is insensitive to prism tilting.

Phase-conjugate resonator (PCR)

A PCR depicted in Fig.5a is composed of a real mirror (RM) with curvature radius R_1 and a phase-conjugate mirror (PCM) formed by a nonlinear optical pheomenon, for example, by degenerate four-wave mixing. The ray transfer matrix $M_{PCM,I}$ and equivalent transfer matrix $M_{PCM,II}$ of PCM are[6,20]

$$M_{PCM,I} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{56}$$

$$M_{PCM,II} = \begin{pmatrix} 1 & 0 \\ -2/p_{PCM} & 1 \end{pmatrix} \tag{57}$$

respectively, where p_{PCM} is the curvature radius of Gaussian beam incident on PCM. $M_{PCM,I}$, $M_{PCM,II}$ are equivalent in calculating beam parameters.

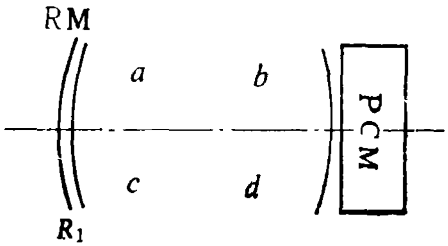

Fig. 5a A phase-conjugate resonator

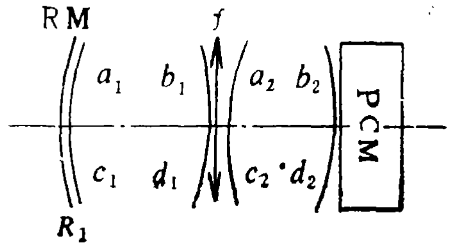

Fig. 5b A phase-conjugate resonator with a thermal lens

Under consideration of the self-consistency condition and the weak-Gaussian aperture approximation, the beam parameters of PCR, i. e. the spot sizes W_{RM}, W_{PCM} and curvature radii p_{M}, p_{PCM} on the RM and PCM are deduced as

$$W_{RM}^2 = \frac{\lambda}{\pi}\left|\frac{b}{G_1}\right| \tag{58}$$

$$W_{PCM}^2 = \frac{2\lambda\,|bG_1|}{\pi} \tag{59}$$

$$p_{RM} = -R_1 \tag{60}$$

$$p_{PCM} = \frac{2bG_1}{2dG_1 - 1} \tag{61}$$

where b, d are the elements of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ inside PCM, G_1 is the resonator G parameter defined by(2).

Introducing
$$G_2 = d - b/p_{PCM} \tag{62}$$
we have
$$G_1G_2 = 1/2 \tag{63}$$

To discuss dynamic stability of PCR, laser rod is regarded as a thermal lens with variable focal length f shown in Fig.5b and the matrix inside the PCR can be rewritten as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \tag{64}$$

Inserting (2), (58), (64) into the condition (18) derivative calculations lead to[21]

$$b_2 = 0 \tag{65}$$

From (63), (65) we conclude that a PCR is dynamic stable, only on condition that thermal disturbance is located in the vicinity of PCM, what differs essentially from conventional dynamic stable resonators, where the thermal lens should be located in the proximity of the output mirror to satisfy the condition(63)

The 4×4 augmented matrix of PCM and RM are

$$T_{PCM} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{66}$$

$$T_{RM} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2/R_1 & 1 & 0 & -2\alpha_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{67}$$

where α_1 is the tilting angle of RM.

Using (58), (59), (66), (67) and the definition, the misalignment sensitivity of PCR was studied by Wang[15], the results are

$$D_1{}^2 = \frac{\pi}{2\lambda}\left|\frac{b}{a-b/R_1}\right| \tag{68}$$

$$D_2 = 0 \tag{69}$$

It means that PCM is an insensitive element against mirror tilting and the misalignment sensitivity of PCR is lower than that of conventional resonators except for

$$R_1 \cong b/a \tag{70}$$

·简　　讯·

第五届全国固体光学性质会议在延吉召开

1990年8月长春物理所、吉林大学、延边科协和中国物理学会在延吉召开第五届全国固体光学性质会议，主要内容是固体非线性、双稳态及其应用，人工晶体、液晶薄膜材料等光学性质研究。

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