激光技术
LASER TECHNOLOGY
Vol.14,No.3

Matrix methods for analysing optical resonators (Part 1)

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Abstract: This paper summarizes some new developments in the application of matrix methods to analysing optical resonators. It has been shown that the important characteristics of resonators i. e., beam parameters, misalignment sensitivity, can be expressed in terms of *ABCD* matrix elements or resonator *g* parameters in general. Some typical resonators such as telescopic resonator with a thermal lens (TRTL), crossed-prism resonator (CPR), phaseconjugate resonator (PCR), and confocal multielement unstable resonator (CMUR) are investigated in detail.

分析光学谐振腔的矩阵方法*(1)

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摘要： 本文总结了将矩阵方法用于研究光腔问题的一些新的进展情况。证明了在普遍情况下描述光腔主要特征的光束参数、失调灵敏度等都能表示为*ABCD*矩阵元或光腔*g*参数的形式。对一些典型的光腔，例如含热透镜的望远镜腔（TRTL）、交叉棱镜腔（CPR）、相位共轭腔（PCR）和共焦型多元件非稳腔（CMUR）等作了详细的分析和讨论。

Introduction

The ray transfer matrix and the *ABCD* law introduced by Kogelnik et al. [1,2] have played an important role in analysing optical resonators since the middle 1960s. It was shown that the beam properties of stable resonators such as spot sizes on the mirrors, waist radius and its location,

*第一届国际光束变换会议（IWBT'89，杭州）报告。

beam divergence angle, and stability, as well as the characteristics of spherical eigenwave of unstable resonators, i. e. location of conjugate points, magnification, diffraction coupling, and unstable condition can be expressed by the resonator *ABCD* matrix. To calculate multielement stable resonators, some equivalent resonators (g', g'', and g^* parameter resonators) based on matrix formalism and imaging properties were presented by Kogelnik, Weber et al.[1,3]. In treating multielement unstable resonators, a canonical formulation was given by Siegman [4]. The applications of those methods and the relating basic concepts can be found in [5~8]. This paper is aimed at summarizing some new developments in studying optical resonators by means of matrix methods reported, especially, in Chinese literatures. Two important concepts concerning dynamic characteristics of resonators, i.e. the dynamic stability and misalignment sensitivity are extended to the general case. Some typical resonators applied in practice, including telescopic resonator with a thermal lens (TRTL), crossed-prism resonator (CPR), and confocal multielement unstable resonator (CMUR) are investigated in detial for illustrating the applications of the methods, and some useful results for resonator design are obtained.

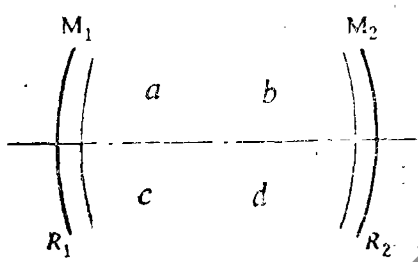

Fig.1 A multielement resonator

Matrix formulation of stable resonators

Fig.1 denotes a typical multielement resonator which is formed by two mirrors M_1, M_2 with radius R_1, R_2, respectively, the matrix inside the resonator is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then the transfer matrix M per round trip referred to M_1 is

$$M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ -2/R_1 & 1 \end{pmatrix}\begin{pmatrix} d & b \\ c & a \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -2/R_2 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{1}$$

Introducing the resonator G parameters G_1, G_2 in the general case as follows

$$G_1=a-\frac{b}{R_1}, \qquad G_2=d-\frac{b}{R_2} \tag{2}$$

(1) can be written as

$$M=\begin{pmatrix} 2aG_2-1 & 2bG_2 \\ 2(2aG_1G_2-G_1-a^2G_2)/bG_1 & 4G_1G_2-2aG_2-1 \end{pmatrix} \tag{3}$$

The self-consistency condition of the q parameter

$$q_1 = \frac{Aq_1 + B}{Cq_1 + D} \tag{4}$$

and (2) (3) lead to

$$W_1^2 = \frac{\lambda B}{\pi\sqrt{1-(A+D)^2/4}} = \frac{\lambda b}{\pi}\sqrt{\frac{G_2}{G_1(1-G_1G_2)}} \tag{5}$$

$$P_1 = 2B/\ (D-A)\ = -R_1 \tag{6}$$

where

$$\frac{1}{q_1} = \frac{1}{p_1} - i\,\frac{\lambda}{\pi W_1{}^2} \tag{7}$$

and W_1, P_1 are the spot size and curvature radius of Gaussian beam on the mirror M_1, respectively, λ is the laser wavelength.

The waist radius near M_1 and its location L_{01}(referred to M_1) are derived by using the $ABCD$ law and (5), (6) as

$$\begin{aligned} W_{01}{}^2 &= -\frac{\lambda}{\pi C}\sqrt{1-\left(\frac{A+D}{2}\right)^2} \\ &= \frac{\lambda b}{\pi}\,\frac{\sqrt{G_1G_2(1-G_1G_2)}}{(G_1+a^2G_2-2aG_1G_2)} \end{aligned} \tag{8}$$

$$L_{01} = \frac{D-A}{2C} = \frac{bG_2(a-G_1)}{(G_1+a^2G_2-2aG_1G_2)} \tag{9}$$

Similar way delivers the spot size W_2 and curvature radius P_2 of Gaussian beam on the mirror M_2, the waist radius W_{02} near M_2 and its location L_{02} (referred to M_2) as

$$W_2{}^2 = \frac{\lambda B'}{\pi\sqrt{1-(A'+D')/4}} = \frac{\lambda b}{\pi}\sqrt{\frac{G_1}{G_2(1-G_1G_2)}} \tag{10}$$

$$P_2 = 2B'/\ (D'-A')\ = -R_2 \tag{11}$$

$$W_{02}{}^2 = -\frac{\lambda}{\pi C'}\sqrt{1-\left(\frac{A'+D'}{2}\right)^2} = \frac{\lambda b}{\pi}\,\frac{\sqrt{G_1G_2(1-G_1G_2)}}{(G_2+d^2G_1-2dG_1G_2)} \tag{12}$$

$$L_{02} = \frac{D'-A'}{2C'} = \frac{bG_1(d-G_2)}{(G_2+d^2G_1-2dG_1G_2)} \tag{13}$$

where A', B', C', D' are the round-trip matrix elements referred to M_2, and the basic relation holds

$$A+D = A'+D' = 2(2G_1G_2-1) \tag{14}$$

The stability condition can be expressed either by $ABCD$ matrix elements or by G parameters

$$\left|\frac{A+D}{2}\right| < 1 \qquad 0 < G_1G_2 < 1 \tag{15}$$

This completes the matrix analysis for multielement stable resonators except for the beam propagation between W_{01} and W_{02}, which can be studied by introducing "hybrid" equivalent resonators, details are found in [9].

General condition for dynamic stability

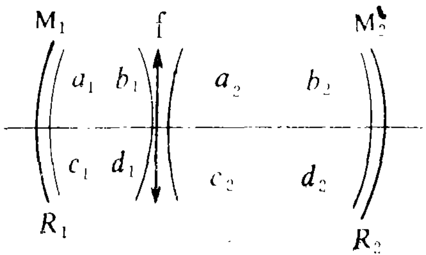

Fig. 2 A generalized resonator with an internal thermal lens

In order to compensate the lensing effects induced by optical pumping in solid-state lasers, dynamic stable resonator was presented by Steffen et al.[10]. In the following we would like to discuss this problem more generally [11].

A generalized resonator with an internal thermal lens of focal length f is sketched in Fig. 2. Assuming that the transfer matrixes between the mirror M_i ($i=1, 2$) and the thermal lens are $\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}$ and $\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}$, the single-pass matrix inside the resonator reads

$$m=\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \tag{16}$$

where

$$\begin{aligned} a&=a_1 a_2+b_2 c_1-a_1 b_2/f \\ b&=a_2 b_1+b_2 d_1-b_1 b_2/f \\ c&=a_1 c_2+c_1 d_2-a_1 d_2/f \\ d&=b_1 c_2+d_1 d_2-b_1 d_2/f \end{aligned} \tag{17}$$

If the output mirror M_1 is adapted, then the condition for dynamic stability can be written as[10]

$$\frac{dW_1}{df}=0 \tag{18}$$

where W_1 is the spot size on the mirror M_1.

Substituting (5) (2) (17) into (18) and considering the basic relation of the transfer matrix $a_i d_i - b_i c_i = 1$ (19)

straightforward derivative calculations yield

$$\frac{1}{G_1}=2G_2+2\left(\frac{b_1}{b_2}\right)+\frac{1}{G_1}\left(\frac{b_1}{b_2}\right)^2 \tag{20}$$

where b_i is the effective length between the mirror M_i and the thermal lens.

Eq. (20) is a general condition for dynamic stability of multielement resonator and can be regarded as an extended expression of (21) in[10]. In comparision with (21) [10], (20) is applicable to more general cases, an example will be shown in Sec. 5.

Misalignment sensitivity

In practice resonator misalignment can't be avoided because of mechanical perturbation, thermal distortion etc.To describe the characteristics of misaligned resonators, an useful parameter——misalignment sensitivity for empty stable resonators was introduced by Weber et al.[13],its definition is as follows.

If both mirrors M_1, M_2 are limited by an adapted aperture and r_{ii}, r_{ji} are the displacements of the intensity patterns at the mirrors M_i, M_j, respectively, and the mirror M_i is tilted by an angle α_i (i,j=1,2 i≠j) then the misalignment sensitivities D_i and D are

$$D_i^2 = \frac{1}{\alpha_i^2}\left[\left(\frac{r_{ii}}{W_i}\right)^2 + \left(\frac{r_{ji}}{W_j}\right)^2\right] = \frac{\pi L}{\lambda}\left(\frac{g_j}{g_i}\right)^{1/2} \frac{1+g_1 g_2}{(1-g_1 g_2)^{3/2}} \tag{21}$$

$$D^2 = D_1^2 + D_2^2 \tag{22}$$

where L is the effective length of resonator and g_1, g_2 are the g parameters defined as

$$g_i = 1 - \frac{L}{R_i} \tag{23}$$

and W_1, W_2 are the spot sizes on the mirrors M_1, M_2, respectively.

For multielement resonators, some lengthy geometrical calculations deliver[14]

$$D_i^2 = \frac{\pi b}{\lambda}\left(\frac{G_j}{G_i}\right)^{1/2} \frac{1+G_1 G_2}{(1-G_1 G_2)^{3/2}} \tag{24}$$

the symbol meaning in (24) is the same as in Sec. 2.

Physically, the larger the parameter D_i is, the larger the additional losses due to the mirror misalignment would be, and the resonator is more sensitive to mirror tilting,so we can compare characteristics of misaligned resonators with the help of parameter D_i.

For the other cases, where only one limiting aperture is located at the mirror or at a distance from the mirror, the results are compiled in [14]. Note that the displacements r_{ii} r_{ji} can be also calculated by using a 4×4 augmented matrix introduced by wang[6], which is convenient for some applications[15,16].